1.5 INEQUALITIES

Solving inequalities حلول المتباينات

is the process of finding the set of numbers that make the inequality a true statement.

- These numbers are called the ***solutions*** of the inequality and we say that they ***satisfy*** the inequality.
- The set of all solutions is called the ***solution set*** of the inequality.
- Inequalities that have the same solution set are said to be ***equivalent***.

Addition and Subtraction Properties

- If $a > b$ and c is a real number, then $a > b$, $a + c > b + c$, and $a - c > b - c$ have the same solution set.
- If $a < b$ and c is a real number, then $a < b$, $a + c < b + c$, and $a - c < b - c$ have the same solution set.

Multiplication and Division Properties

- If $c > 0$ the inequalities $a > b$, $ac > bc$, and $\frac{a}{c} > \frac{b}{c}$ have the same solution set.
- ★ If $c < 0$ the inequalities $a > b$, $ac < bc$, and $\frac{a}{c} < \frac{b}{c}$ have the same solution set.

Ex.1. Solve the inequality $3 - 2x < 11$

Solution:

$3 - 2x - 3 < 11 - 3$ Subtract 3 from both sides.

$-2x < 8$ Simplify

$$\frac{-2x}{-2} > \frac{8}{-2}$$

$x > -4$ The solution

$\{x | x > -4\}$ Expressed in set builder notation

$(-4, \infty)$ Expressed in interval notation

1. Compound inequality المتباينات المركبة

Is an inequality formed by joining two inequalities with “and” or “or.”

Ex.2. Solve $x + 2 < 5$ **and** $2x - 6 > -8$.

Solution:

Solve the first inequality.

$x + 2 < 5$

$x < 3$ Subtract 2.

$\{x \mid x < 3\}$ Solution set

Solve the second inequality.

$2x - 6 > -8$

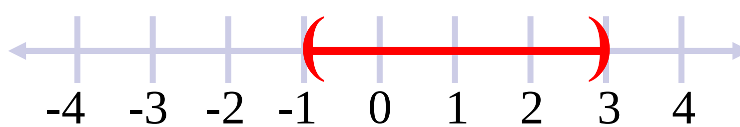
$2x > -2$ Add 6.

$x > -1$ Divide by 2.

$\{x \mid x > -1\}$ Solution set

$$\{x \mid x < 3\} \cap \{x \mid x > -1\} = \{x \mid -1 < x < 3\}$$

The solution set of the "and" compound inequality is the *intersection* of the two solution sets.



Ex.3. Solve $-3 < 2x + 1 \le 3$

Solution:

$$-3 - 1 < 2x + 1 - 1 \le 3 - 1$$ Subtract 1

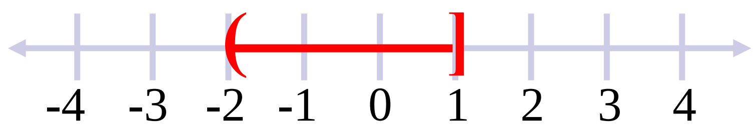
$$-4 < 2x \le 2$$ Simplify.

$$\frac{-4}{-2} < \frac{2x}{2} \le \frac{2}{2}$$

$$-2 < x \le 1$$ The solution

$$\{x \mid -2 < x \le 1\}$$ Set builder notation.

$$(-2, 1]$$ Interval notation



Note: The inequality $-3 < 2x + 1 \le 3$ can be written in the form $-3 < 2x + 1$ and $2x + 1 \le 3$

Ex.4. Solve $x + 5 > 6$ **or** $2x < -4$.

Solution:

Solve the first inequality.

$$x + 5 > 6$$

$$x > 1$$

$\{\, x \mid x > 1 \}$ Solution set

Solve the second inequality.

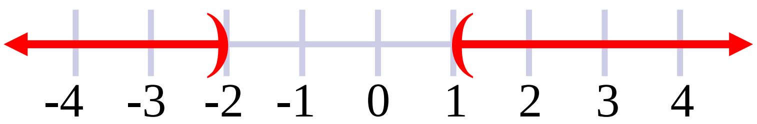
$$2x < -4$$

$$x < -2$$

$\{\, x \mid x < -2 \}$ Solution set

$$\{x \mid x > 1\} \cup \{x \mid x < -2\}$$

Since the inequalities are joined by "or" the solution set is the *union* of the solution sets.



Ex. Solve the following Inequalities:

1. $4x < -8 \ \ and \ \ 1 - 2x > 5$

2. $-4x + 5 > 9 \ \ or \ \ 4x + 1 < 5$

3. $0 \le 2x + 6 \le 54$

2. Absolute Value Inequality متباينة القيمة المطلقة

If X is an algebraic expression and k is a nonnegative number,

1. $|X| < k \quad \text{if and only if} \quad -k < X < k.$

1. $|X| > k \quad \text{if and only if} \quad X < -k \ \text{ or } \ X > k$

Rules are valid if < is replaced by ≤
and > is replaced by ≥.

Ex.5. Solve the inequality $|x - 4| < 3$

Solution:

$|x - 4| < 3$ means $-3 < x - 4 < 3$

$$-3 + 4 < x - 4 + 4 \leq 3 + 4$$

$1 < x < 7$ Solution

$\{x|\ 1 < x < 7\}$ Set builder notation

$(1, 7)$ Interval notation.

Ex.6. Solve the inequality $|2x + 3| \geq 5$

Solution: $|2x + 3| \geq 5$ means $2x + 3 \leq -5$ or $2x + 3 \geq 5$

$2x + 3 - 3 \leq -5 - 3$ $\quad$ $2x + 3 - 3 \geq 5 - 3$

$2x \leq -8$ $\quad$ $2x \geq 2$

$x \leq -4$ $\quad$ $x \geq 1$

$\{x|\ x \leq -4$ or $x \geq 1\}$ Set builder and $(-\infty, -4] \cup [1, \infty)$ Interval notation

Note: The solution of the following inequalities are straight forward:

1. $|x-5|<0$ → $No\,Solution$
2. $|x-5|<-2$ → $No\,Solution$
3. $|x-5|\leq 0$ → $x=5$
4. $|x-5|>-3$ → $(-\infty,\infty)$
5. $|x-5|>0$ → $(-\infty,5)\cup(5,\infty)$
6. $|x-5|\geq 0$ → $(-\infty,\infty)$

Ex. Solve the following Inequalities:

1. $|3x - 10| \leq 14$

2. $|4 - 5x| \geq 24$

3. $|5 - 6x| \geq 0$

4. $|6x - 9| < -2$

5. $0 < |x - 5| < 2$

3. Critical Value Method

We use this method to solve: i) Polynomial Inequalities

ii) Rational Inequalities

Procedure for Solving Inequalities

1. Express the inequality such that one side is *zero*
2. Find the real zeros of the of the numerator and denominator. These zeros are the critical values of the inequality.
3. Locate these critical values on a number line, thereby dividing the number line into ***test intervals.***
4. Choose one representative number within each test interval.
   a) If substituting that value into the original inequality produces a true statement then all real numbers in the test interval belong to the solution set.

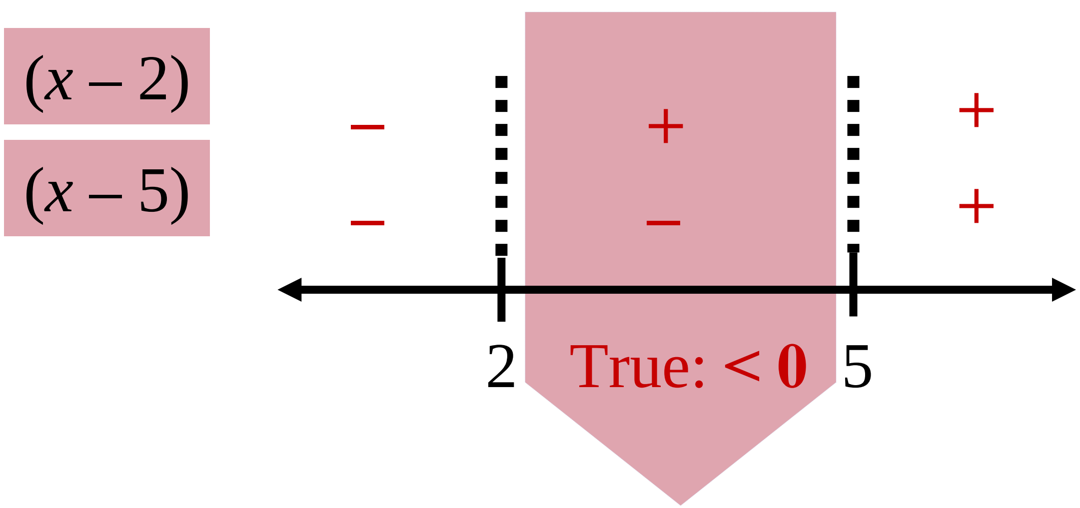
b) If substituting that value into the original inequality produces a false statement, then no real numbers in the test interval belong to the solution set

5. Write the solution set, selecting the interval(s) that produced a true statement.

Ex.7. Solve the inequality $x^2 - 7x + 10 < 0$

Solution: $(x - 2)(x - 5) < 0$ Factor

$x = 2$ and $x = 5$ Critical Values



solution set is the interval (2, 5).

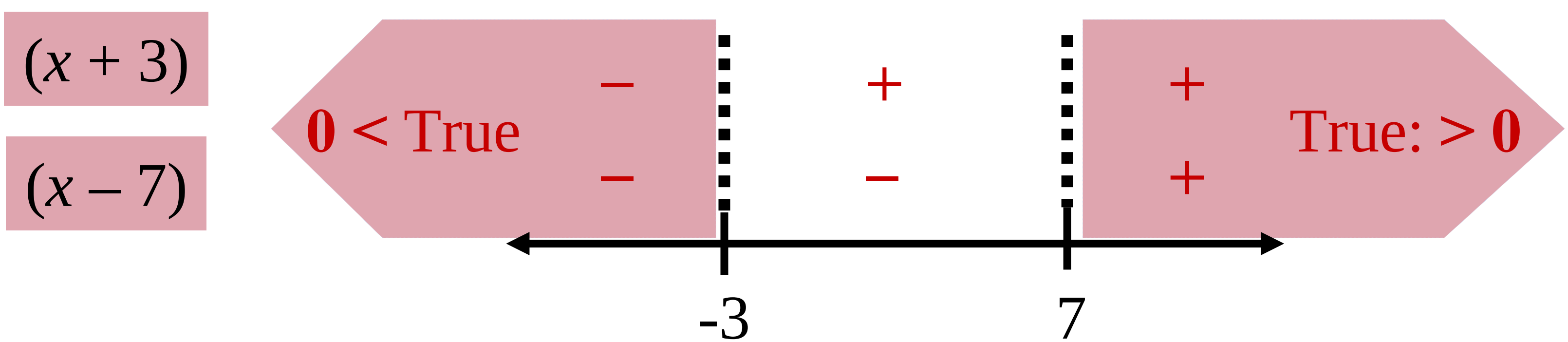
Note: Don’t multiply the inequality by an algebraic expression

Ex.8. Solve the inequality $\dfrac{x+3}{x-7} > 0$

Solution:

$x = -3$ and $x = 7$

The Critical Values: Set the numerator and denominator equal to zero



Solution set is the interval $(-\infty, -3) \cup (7, \infty)$.

Ex.9. Solve the inequality $\dfrac{x+1}{x+3} \le 2$

Solution:

$$\frac{x+1}{x+3} - 2 \le 0$$

One side zero

$$\frac{x+1}{x+3} - \frac{2(x+3)}{x+3} \le 0$$

Simplify by taking the LCD

$$\frac{x+1-2(x+3)}{x+3} \le 0$$

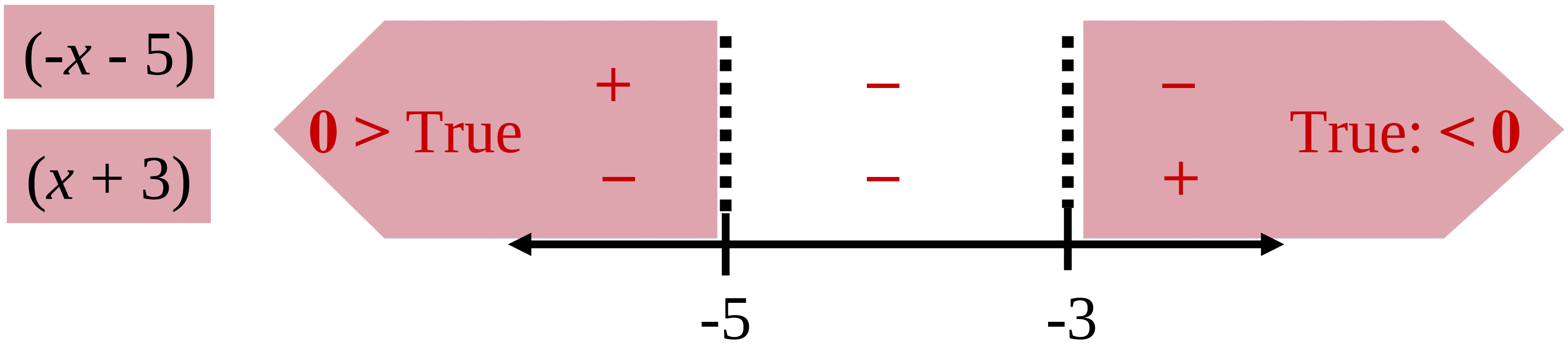
Simplify

$$\frac{-x-5}{x+3} \le 0$$

Set the numerator and denominator equal to zero

$x = -5$ and $x = -3$

The Critical Values:



Solution set is the interval $(-\infty, -5] \cup (-3, \infty)$

-3 is not included
It is the zero of the denominator

Ex. Solve the following Inequalities:

1. $x^2 < -x + 30$

2. $x^2 + 7x + 10 > 0$

3. $\dfrac{x-2}{x+3} > 0$

4. $\dfrac{1}{x} \le x$

5. $\dfrac{x+2}{x-5} \le 2$

6. $\dfrac{x^2 + 10x + 25}{x+1} \ge 0$

7. $\dfrac{(x-4)^2}{(x+3)^3} \ge 0$

8. $\dfrac{(x-4)^2}{(x+3)^2} \ge 0$