MATH 001 Review Problems On Equations And Functions

1. Find the domain and range of the following functions :
   (a) $f(x)=\sqrt{\frac{x-1}{x}}$ (D $=(-\infty,0)\cup[1,\infty)$; R $=[0,1)\cup(1,\infty)$)
   (b) $s(x)=\frac{-1}{x^2+6x+9}$ (D $=(-\infty,-3)\cup(-3,\infty)$; R $=(-\infty,0)$)
   (c) $g(x)=-\sqrt{49-x^2}$ (D $=[-7,7]$; R $=[-7,0]$)
   (d) $h(x)=\sqrt[3]{x^2-8}$ (D $=(-\infty,\infty)$; R $=[-2,\infty)$)
   (e) $t(x)=-\sqrt{x^2+6x+5}$ (D $=(-\infty,-5]\cup[-1,\infty)$; R $=(-\infty,0]$)
   (f) $k(x)=\sqrt{\frac{1}{x^2+4}}$ (D $=(-\infty,\infty)$; R $=(0,\frac{1}{2}]$)
   (g) $r(x)=-|x-5|+2$ (D $=(-\infty,\infty)$; R $=(-\infty,2]$)

2. If $f(x)=[[2x+4]]$, then find the solution set of $f(x)=2$ and $f(x)=-3$, and graph $y=f(x)$. (S.S.$=[-1,-\frac{1}{2})$, S.S.$=[-\frac{7}{2},-3)$)

3. If $f(x)=\frac{1}{x-4}$, $g(x)=\sqrt{x-2}$, find the domain of $(f+g)(x)$ and the value of $(f+g)(11)$. (D $=[2,4)\cup(4,\infty)$; $\frac{22}{7}$)

4. If $f(x)=\begin{cases}[[2x]] & \text{if } x\geq 1\\ 2x-3 & \text{if } x<1\end{cases}$, find $(f\circ f)(-0.4)\,,(f\circ f)(3.6)$. $(-10.6\,;\,14)$

5. Find the domain of $(\frac{f}{g})(x)$, where $f(x)=\sqrt{x+4}$, $g(x)=\frac{\sqrt{36-x^2}}{x+2}$. (D $=[-4,-2)\cup(-2,6)$)

6. Find the sum of all solutions of the equation $8x^3+27=0$. (0)

7. If we rewrite the equation $3x^2-4x-1=0$ as $(x-m)^2=n$, then find $m+n$. $(\frac{13}{9})$

8. Find all values of k for which the equation $9x^2+(k+1)x+4=0$ has two equal real roots. ($k=11$ or $k=-13$)

9. Find all values of k for which the equation $(k+6)x^2+2kx+1=0$ has two non-real complex roots. ($k\in(-2,3)$)

10. Find the number of real solutions for the equations:
    (a) $\sqrt{x+5}-\sqrt{2x-4}=1$ (b) $\sqrt[4]{x-1}-\sqrt{x}=0$ (1 ; 0)

11. Find the product of all real solutions of the equation $\sqrt[4]{5x^2-6}=x$ $(\sqrt{6})$

12. If m and n are the roots ofthe equation $2x^2+3x+8=0$, then find the equations whose roots are
    (a) 2m and 2n (b) $-m,-n$ (c) $\frac{1}{m}$, $\frac{1}{n}$. $(x^2+3x+16=0\,;2x^2-3x+8=0\,;\,8x^2+3x+2=0)$

13. Solve the inequality $x^2+4x+5>0$ (S.S. $=(-\infty,\infty)$)

14. Solve the inequality $1<|x^2-1|<3$ (S.S. $=(-2,-\sqrt{2})\cup(\sqrt{2},2)$)

15. Solve the inequality $-2<\frac{1}{x-1}<1$ (S.S. $=(-\infty,\frac{1}{2})\cup(2,\infty)$)

16. Solve the inequality $\frac{(2-x)^3(x-1)^2}{(x^2+3)(x+1)}\leq 0$ (S.S. $=(-\infty,-1)\cup\{1\}\cup[2,\infty)$)

17. If $f(x)=\sqrt{2x+5}$, $g(x)=2x^2-2x-2$, find $(f\circ g)(x)$. ($|2x-1|$)

18. If $(f \circ g)(x) = 4x + 5$, $f(x) = 2x - 1$, find $g(x)$. $(g(x) = 2x + 3)$

19. If the points $(0, a)$, $(2, \frac{2}{3})$, $(-3, 4)$, $(b, 0)$ lie on a straight line, then find $a + b$. $(a + b = 5)$

20. Find the extreme values of : (a) $y = x^2 + x$ (b) $y = x^2 + 4x - 1$, $-5 \leq x \leq 1$
((a) Min. $= -\frac{1}{4}$; (b) Min. $= -5$, Max. $= 4$)

21. Find the distance between the center of the circle $x^2 - 2x + y^2 + 4y = -4$ and the vertex of the parabola $x = -y^2 + 6y - 10$. $(\sqrt{29})$

22. Find the value of a if the line $-2x + 3ay = 1$ is parallel to the line $4x + 9y = 3$. $(a = -\frac{3}{2})$

23. If $x = 2$ is the symmetry axis of the parabola $y = -x^2 + mx - 1$, then find the maximum value.
(Max. $= 3$)

24. Find the equation of the line passing through the points $(k, 3k)$, $(k, k + 4)$. $(x = k)$

25. Find the solution set of $|x + 2|^2 + 3|x + 2| - 10 = 0$. $(\text{S.S.} = \{-4, 0\})$

26. Solve the equation $[[x^2 - 1]] = 1$. $(\text{S.S.} = (-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3}))$

27. Find the x and y – intercepts of $f(x) = \begin{cases} 5 - 2x & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$ $(x = 3\,;\, y = 5)$

28. Find the equation of the perpendicular bisector of the line segment joining the points $(3, -2)$, $(1, -3)$
$(4x + 2y = 3)$

29. Find the y-intercept of the line having x-intercept 2 and perpendicular to $2x - y = 1$. (1)

30. Find the equation of the circle having the points $(2, 3)$, $(-2, 1)$ as endpoints of a diameter.
$(x^2 + y^2 - 4y - 1 = 0)$

31. If the point $(1, 4)$ is 5 units from the midpoint of the segment joining $(3, -2)$ and $(x, 4)$, then find the values of x. $(x = 7\,,\, -9)$

32. Find the range of the function $f(x) = -2x^2 + 3x + 4$ $(\text{Range} = (-\infty, \frac{41}{8}])$

33. Find all values of k for which the graph of the parabola $y = kx^2 + kx + 1$ has two x-intercepts.
$(k \in (-\infty, 0) \cup (4, \infty))$

34. Find the shortest distance between the point $(-2, 1)$ and the line $y = 3x - 3$. $(\sqrt{10})$

35. Is the point $(2, -4)$ on, inside, or outside the circle $x^2 - 6x + y^2 + 4y = -9$? (outside)

36. Which one of the following is a function?
$x^2 + y^2 = 4$; $2y^3 + 4x = 3$; $|x| + 3|y| = 2$; $x = 6$; $x = |y - 3|$ $(2y^3 + 4x = 3)$

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