Math 001 — Real Numbers

Let :
N = The set of **natural** numbers = $\{1, 2, 3, 4, ...\}$.
W = The set of **whole** numbers = $\{0, 1, 2, 3, ...\}$.
Z = The set of **integer** numbers = $\{..., -2, -1, 0, 1, 2, ...\}$.
Q = The set of **rational** numbers = $\left\{x | x = \frac{p}{q}; \, p, q \in Z \, , \, q \neq 0\right\}$.
H = The set of **irrational** numbers = $\{x | x \in \Re \, , \, x \notin Q\}$.
$\Re$ = The set of **real** numbers .

Notice that any **integer** , **fraction** , **terminating decimal** or **repeating decimal** number can be written in the form $\frac{p}{q}$, where , $p, q \in Z, q \neq 0$, therefore it is a **rational** number.
For example : $0 = \frac{0}{1}$, $-3 = \frac{-3}{1}$, $-\frac{4}{5}$, $1.23 = \frac{123}{100}$, $0.059 = \frac{59}{1000}$, $1.3333... = \frac{4}{3}$, and $0.262626... = \frac{26}{99}$ all are **rational** numbers . The numbers π , $\frac{2\pi}{5}$, $\frac{\sqrt{8}}{3}$,$\sqrt[3]{2}$ and 5.204781.....all are **irrational** numbers , while , $\frac{7}{0}$, $\sqrt{-9}$ and $\sqrt[4]{-16}$ are **not real** numbers .
Notice that $\frac{22}{7}$ and $3.14 = \frac{314}{100}$ are **rational** numbers while π is an **irrational** number .This is because $\pi \neq \frac{22}{7}$ or 3.14 but $\pi \approx \frac{22}{7}$ or 3.14 . In fact $\boxed{3.14 < \pi < \frac{22}{7}}$

We can notice the following:

- $N \subset W \subset Z \subset Q \subset \Re$ **and** $H \subset \Re$.
- $Q \bigcup H = \Re$ **and** $Q \bigcap H = \emptyset$.
- The sets N , W , Z, Q and $\Re$ all are **closed** under **addition** and **multiplication** .
- Only the sets Z , Q and $\Re$ are **closed** under **subtraction** , e.g. , 2 , 5 $\in N$ but $2-5 = -3 \notin N$.
- None of the above sets is **closed** under **division** , e.g. , 4 , 7 $\in N$ but $\frac{4}{7} \notin N$,similarly for W and Z . Also 0.5 , 0 $\in Q, \Re$ but $\frac{0.5}{0} \notin Q, \Re$.
- The set H is **not closed** under any operation , e.g. ,$\sqrt{3}$, $-\sqrt{3} \in H$ but $\sqrt{3} + (-\sqrt{3}) = 0 \notin H$; $\sqrt{3} - \sqrt{3} = 0 \notin H$; $\sqrt{3} . \sqrt{3} = 3 \notin H$ and $\frac{\sqrt{3}}{\sqrt{3}} = 1 \notin H$.

  Let us summarize all of this in the following table :

Table 1: Closure − Property

Set	+	−	×	÷
N	yes	no	yes	no
W	yes	no	yes	no
Z	yes	yes	yes	no
Q	yes	yes	yes	no
H	no	no	no	no
$\Re$	yes	yes	yes	no

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Example : The set $A = \{-1, 0\}$ is **not** closed under addition for : $(-1)+(-1) = -2 \notin A$, **not** closed under subtraction for : $0-(-1) = 1 \notin A$, **not** closed under multiplication for : $(-1)(-1) = 1 \notin A$ and **not** closed under division for : $\frac{-1}{-1} = 1 \notin A$.

- All the sets N , W , Z , Q , H and $\Re$ satisfy the **commutative** property for addition and multiplication , i . e . , $a+b = b+a$ & $ab = ba$, whenever a, b are in the set .
- All the above sets satisfy the **associative** property for addition and multiplication , i . e . , $(a+b)+c = a+(b+c)$ & $(ab)c = a(bc)$, whenever a, b, c are in the set .
- The sets $\Re$, Q , Z and W contain the **identity** element for addition (0) and the **identity** element for multiplication (1) . The set N contains only the **identity** element for multiplication while the set H **does not** contain any one of them .
- For any **real** number a , $(-a)$ is called the **additive inverse** of a which satisfies : $a+(-a) = (-a)+a = 0$.
- For any **nonzero real** number a , $\frac{1}{a}$ is called **multiplicative inverse** of a which satisfies : $a.(\frac{1}{a}) = (\frac{1}{a}).a = 1$.
  Next let us find the add. and mult. inverse of some real numbers in the following table :

Table 2: Inverse − Property

No.	Add. Inv.	Mul. Inv.
-1	1	-1
0	0	dne
$\frac{-3}{2}$	$\frac{3}{2}$	$\frac{-2}{3}$
$\frac{2}{\pi}$	$\frac{-2}{\pi}$	$\frac{\pi}{2}$
-0.04	0.04	-25

Finally we like to name the property for each of the following statements :

1) $5+(-5) = (-5)+5$: comm. prop. of add.
2) $5+(-5) = 0$: inv. prop. of add.
3) $5+0 = 5$: iden. prop. of add.
4) $[5+(-5)]+0 = 5+[(-5)+0]$: assoc. prop. of add.
5) $5.(\frac{1}{5}) = (\frac{1}{5}).5$: comm. prop. of mult.
6) $5.(\frac{1}{5}) = 1$: inv. prop. of mult.
7) $5.1 = 5$: iden. prop. of mult.
8) $(5).(0) = 0$: prop. of zero.

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