Math 001 Rationalizing The Denominator

- To rationalize the denominator of a rational expression we must get rid of all radicals in the denominator such as $\sqrt{x-1}$, $\sqrt[3]{2}$, $\sqrt[4]{y^2+2}$ etc. To do so, we must remember the following :

1) $(\sqrt[n]{x})^n = x$ (for $x \geq 0$ when n is a positive even integer or for $x \in \Re$ when n is a positive odd integer). For example $(\sqrt{5})^2 = 5$, $(\sqrt[3]{x-3})^3 = x-3$, $(\sqrt[4]{y+2})^4 = y+2$.

2) $(x-y)(x+y) = x^2 - y^2$, so $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3-2 = 1$ and $(\sqrt{x+3}-\sqrt{x-3})(\sqrt{x+3}+\sqrt{x-3}) = (x+3)-(x-3) = 6$.

3) $(x+y)(x^2-xy+y^2) = x^3+y^3$ and $(x-y)(x^2+xy+y^2) = x^3-y^3$. For example
$(\sqrt[3]{x}-\sqrt[3]{y})(\sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}) = (\sqrt[3]{x})^3 - (\sqrt[3]{y})^3 = x-y$ and
$(\sqrt[3]{m}+\sqrt[3]{4})(\sqrt[3]{m^2}-\sqrt[3]{4m}+\sqrt[3]{16}) = (\sqrt[3]{m})^3 + (\sqrt[3]{4})^3 = m+4$.

4) For $x, y \geq 0$, $(\sqrt[4]{x}-\sqrt[4]{y})(\sqrt[4]{x}+\sqrt[4]{y}) = (\sqrt[4]{x})^2 - (\sqrt[4]{y})^2 = \sqrt[4]{x^2}-\sqrt[4]{y^2} = \sqrt{x}-\sqrt{y}$.

- Now we are ready and able to rationalize the denominators of the following rational expressions :

1) $\frac{\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2}\cdot\sqrt{3}}{5(\sqrt{3})^2} = \frac{\sqrt{6}}{5\cdot 3} = \frac{\sqrt{6}}{15}$. ; $\frac{\sqrt[3]{3}}{\sqrt[3]{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}} \cdot \frac{(\sqrt[3]{2})^2}{(\sqrt[3]{2})^2} = \frac{\sqrt[3]{3}\cdot\sqrt[3]{4}}{(\sqrt[3]{2})^3} = \frac{\sqrt[3]{12}}{2}$.

$\frac{\sqrt[5]{x}}{\sqrt[4]{x}} = \frac{\sqrt[5]{x}}{\sqrt[4]{x}} \cdot \frac{(\sqrt[4]{x})^3}{(\sqrt[4]{x})^3} = \frac{\sqrt[5]{x}\cdot(\sqrt[4]{x})^3}{(\sqrt[4]{x})^4} = \frac{x^{\frac{1}{5}}\, x^{\frac{3}{4}}}{x} = \frac{x^{\frac{19}{20}}}{x} = \frac{\sqrt[20]{x^{19}}}{x}$.

2) $\frac{\sqrt{3}}{\sqrt{2}-1} = \frac{\sqrt{3}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{3}\cdot(\sqrt{2}+1)}{2-1} = \sqrt{6}+\sqrt{3}$.

3) $\frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}+\sqrt{x+2}} = \frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}+\sqrt{x+2}} \cdot \frac{\sqrt{x}-\sqrt{x+2}}{\sqrt{x}-\sqrt{x+2}} = \frac{(\sqrt{x}-\sqrt{x+2})^2}{(\sqrt{x})^2-(\sqrt{x+2})^2} = \frac{x-2\sqrt{x}\cdot\sqrt{x+2}+x+2}{x-(x+2)}$
$= \frac{2x-2\sqrt{x^2+2x}+2}{-2} = -(x-\sqrt{x^2+2x}+1)$.

4) $\frac{1}{(\sqrt{3}-\sqrt{2})\,(\sqrt{x}+1)} = \frac{1}{(\sqrt{3}-\sqrt{2})\,(\sqrt{x}+1)} \cdot \frac{(\sqrt{3}+\sqrt{2})\,(\sqrt{x}-1)}{(\sqrt{3}+\sqrt{2})\,(\sqrt{x}-1)} = \frac{\sqrt{3x}+\sqrt{2x}-\sqrt{3}-\sqrt{2}}{(3-2)\,(x-1)}$
$= \frac{\sqrt{3x}+\sqrt{2x}-\sqrt{3}-\sqrt{2}}{x-1}$.

5) $\frac{6}{\sqrt[3]{5}-\sqrt[3]{2}} = \frac{6}{\sqrt[3]{5}-\sqrt[3]{2}} \cdot \frac{\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4}}{\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4}} = \frac{6(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})}{(\sqrt[3]{5})^3-(\sqrt[3]{2})^3} = \frac{6(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})}{5-2}$
$= \frac{6(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})}{3} = 2(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})$.

6) $\frac{\sqrt[3]{4}}{\sqrt[3]{x^2}-\sqrt[3]{2x}+\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{x^2}-\sqrt[3]{2x}+\sqrt[3]{4}} \cdot \frac{\sqrt[3]{x}+\sqrt[3]{2}}{\sqrt[3]{x}+\sqrt[3]{2}} = \frac{\sqrt[3]{4}\cdot(\sqrt[3]{x}+\sqrt[3]{2})}{(\sqrt[3]{x})^3+(\sqrt[3]{2})^3} = \frac{\sqrt[3]{4x}+\sqrt[3]{8}}{x+2} = \frac{\sqrt[3]{4x}+2}{x+2}$.

7) $\frac{1}{\sqrt[4]{3}-1} = \frac{1}{\sqrt[4]{3}-1} \cdot \frac{\sqrt[4]{3}+1}{\sqrt[4]{3}+1} = \frac{\sqrt[4]{3}+1}{(\sqrt[4]{3})^2-(1)^2} = \frac{\sqrt[4]{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt[4]{27}+\sqrt[4]{3}+\sqrt{3}+1}{3-1}$
$= \frac{\sqrt[4]{27}+\sqrt[4]{3}+\sqrt{3}+1}{2}$.

BY : A. AL SHALLALI