Math 002. **Quiz # 4** **Sem. II , 27 – 4 – 2005**

Name :____________________________ **ID. # :** ______ **SER. # :** ____

- **Circle the correct answer and show your work**

1. $\sin[2\tan^{-1}(-\frac{5}{12})] =$

   (a) $-\frac{60}{169}$

   (b) $-\frac{65}{144}$

   (c) $\frac{45}{144}$

   (d) $-\frac{120}{169}$

   (e) $-\frac{10}{13}$

2. If $y = -4\sin x + 4\sqrt{3}\cos x = k\cos(x+\alpha)$, then $\alpha =$

   (a) $\frac{\pi}{3}$

   (b) $\frac{3\pi}{4}$

   (c) $\frac{5\pi}{6}$

   (d) $\frac{\pi}{6}$

   (e) $\frac{5\pi}{3}$

3. The value of $(\sin 22.5^\circ \cos 22.5^\circ) + \dfrac{4\tan 67.5^\circ}{1-\tan^2 67.5^\circ}$ is:

   (a) $\dfrac{\sqrt{2}+4}{2}$

   (b) $\dfrac{\sqrt{2}-4}{4}$

   (c) $\dfrac{\sqrt{3}-4}{2}$

   (d) $\dfrac{\sqrt{3}+\sqrt{2}}{2}$

   (e) $\dfrac{\sqrt{2}-8}{4}$

4. Which one of the following is **TRUE**:

(a) The domain of $y = 2\cos^{-1}(4-3x)$ is $[-\frac{1}{3}, \frac{4}{3}]$

(b) $\sin(\sin^{-1} 2x) = 2x$ if $-1 \le x \le 1$

(c) $\sin^{-1}(\sin\frac{5\pi}{4}) + \cos^{-1}(\cos\frac{5\pi}{4}) = \frac{\pi}{2}$

(d) The range of $y = 2\sin^{-1}(3x+1) + \pi$ is $[-\pi, \frac{3\pi}{2}]$

(e) $\sec^{-1}(-\sqrt{2}) = -\frac{\pi}{4}$

5. The **sum** of all solutions of the equation $\sin^2\frac{x}{2} + \frac{3}{2}\cos x = 0$, $0 \le x \le 3\pi$ is:

(a) $\frac{14\pi}{3}$

(b) $\frac{29\pi}{6}$

(c) $\frac{22\pi}{3}$

(d) $\frac{13\pi}{6}$

(e) $\frac{23\pi}{4}$

See the solution on the next page

The Solution

1)

Let $\alpha = \tan^{-1}(-\frac{5}{12})$
$\Longrightarrow \tan\alpha = -\frac{5}{12}$, $\alpha \in$ Q IV$^-$
$\Longrightarrow x = 12$, $y = -5$, $r = \sqrt{144+25} = 13$, so
$\sin\alpha = -\frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Therefore
$\sin 2\alpha = 2\sin\alpha \ \cos\alpha = 2(-\frac{5}{13})(\frac{12}{13}) = -\frac{120}{169}$

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2)

$a = -4$, $b = 4\sqrt{3} \Longrightarrow k = \sqrt{16+48} = 8$ and
$\cos\beta = \frac{a}{k} = \frac{-4}{8} = -\frac{1}{2}$, $\sin\beta = \frac{b}{k} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$
$\Longrightarrow \beta \in$ Q II , $\beta' = \frac{\pi}{3}$ and so $\beta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.
Therefore $y = 8\sin(x + \frac{2\pi}{3}) = 8\cos[\frac{\pi}{2} - (x + \frac{2\pi}{3})]$
$= 8\cos(-x - \frac{\pi}{6}) = 8\cos(x + \frac{\pi}{6}) \Longrightarrow \alpha = \frac{\pi}{6}$

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3)

$= \frac{1}{2} \cdot (2\sin 22.5^\circ \ \cos 22.5^\circ) + 2 \cdot \frac{2\tan 67.5^\circ}{1 - \tan^2 67.5^\circ}$
$= \frac{1}{2}\sin 45^\circ + 2\tan 135^\circ = \frac{1}{2} \ \frac{\sqrt{2}}{2} + 2(-1) = \frac{\sqrt{2}-8}{4}$

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4)

$\sin^{-1}(\sin\frac{5\pi}{4}) + \cos^{-1}(\cos\frac{5\pi}{4})$
$= \sin^{-1}(-\frac{\sqrt{2}}{2}) + \cos^{-1}(-\frac{\sqrt{2}}{2})$
$= -\sin^{-1}\frac{\sqrt{2}}{2} + (\pi - \cos^{-1}\frac{\sqrt{2}}{2})$
$= -\frac{\pi}{4} + (\pi - \frac{\pi}{4}) = \frac{\pi}{2}$ is True.

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5)

$\Longrightarrow \frac{1-\cos x}{2} + \frac{3}{2}\cos x = 0$, $0 \le x \le 3\pi$
$\Longrightarrow 1 - \cos x + 3\cos x = 0 \Longrightarrow 2\cos x = -1$
$\Longrightarrow \cos x = -\frac{1}{2} \Longrightarrow x \in$ Q II or III , $x' = \frac{\pi}{3}$
So, $x = \frac{2\pi}{3}$, $\frac{4\pi}{3}$ or $2\pi + \frac{2\pi}{3} = \frac{8\pi}{3}$
Sum $= \frac{2\pi}{3} + \frac{4\pi}{3} + \frac{8\pi}{3} = \frac{14\pi}{3}$