Math 002.08 **Quiz # 3** **Sem. II , 3 – 4 – 2005**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

- **Be clean and show your work**

1. Find the <u>vertical asymptotes</u> and the <u>x–intercepts</u> of the graph of $y = -3\tan(2x + \frac{\pi}{2})$ over the interval $[-\pi, \frac{3\pi}{2}]$. (no need to graph) (3 pts)

<u>Ans</u>: The graph has vertical asymptotes when $2x + \frac{\pi}{2} = (2n+1)\frac{\pi}{2} \implies 2x = (2n)\ \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2}$ $= n\ \pi \implies x = n\ \frac{\pi}{2},\ n \in Z$. So over the interval $[-\pi, \frac{3\pi}{2}]$: if $n = -2$, $\boxed{x = -\pi}$; if $n = -1$, $\boxed{x = -\frac{\pi}{2}}$; if $n = 0$, $\boxed{x = 0}$; if $n = 1$, $\boxed{x = \frac{\pi}{2}}$; if $n = 2$, $\boxed{x = \pi}$; if $n = 3$, $\boxed{x = \frac{3\pi}{2}}$

While the graph has x–intercepts when $2x + \frac{\pi}{2} = n\ \pi \implies 2x = n\ \pi - \frac{\pi}{2} = (2n-1)\ \frac{\pi}{2}$ $\implies x = (2n-1)\ \frac{\pi}{4},\ n \in Z$. So over the interval $[-\pi, \frac{3\pi}{2}]$: if $n = -1$, $\boxed{x = -\frac{3\pi}{4}}$;

if $n = 0$, $\boxed{x = -\frac{\pi}{4}}$; if $n = 1$, $\boxed{x = \frac{\pi}{4}}$; if $n = 2$, $\boxed{x = \frac{3\pi}{4}}$; if $n = 3$, $\boxed{x = \frac{5\pi}{4}}$.

Therefore the x–intercepts are $(-\frac{3\pi}{4}, 0)$, $(-\frac{\pi}{4}, 0)$, $(\frac{\pi}{4}, 0)$, $(\frac{3\pi}{4}, 0)$, and $(\frac{5\pi}{4}, 0)$.

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2. If $\sin\alpha = -\frac{4}{5}$, $\pi < \alpha < \frac{3\pi}{2}$ and $\cos\beta = -\frac{12}{13}$, $\frac{\pi}{2} < \beta < \pi$, find

$\tan(\pi + \alpha + \beta)$, $\cos(\beta + \frac{\pi}{4})$, $\cos(\alpha - \beta + \frac{\pi}{2})$ (4 pts)

<u>Ans</u>: α lies in Q III, $\sin\alpha = -\frac{4}{5} = \frac{y}{r} \implies y = -4, r = 5, x = -\sqrt{25 - 16} = -3$

$\implies \cos\alpha = -\frac{3}{5}, \tan\alpha = \frac{4}{3}$ and β lies in Q II, $\cos\beta = -\frac{12}{13} = \frac{x}{r} \implies x = -12, r = 13,$

$y = \sqrt{169 - 144} = 5 \implies \sin\beta = \frac{5}{13}, \tan\beta = -\frac{5}{12}$. So

(i) $\tan(\pi + \alpha + \beta) = \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \dfrac{\frac{4}{3} + (-\frac{5}{12})}{1 - (\frac{4}{3})(-\frac{5}{12})} \cdot \dfrac{36}{36} = \dfrac{48 - 15}{36 + 20} = \dfrac{33}{56}$

(ii) $\cos(\beta + \frac{\pi}{4}) = \cos\beta\ \cos\frac{\pi}{4} - \sin\beta\ \sin\frac{\pi}{4} = (-\frac{12}{13})\ (\frac{\sqrt{2}}{2}) - (\frac{5}{13})\ (\frac{\sqrt{2}}{2}) = -\frac{17\sqrt{2}}{26}$

(iii) $\cos(\alpha - \beta + \frac{\pi}{2}) = \cos[\frac{\pi}{2} - (\beta - \alpha)] = \sin(\beta - \alpha) = \sin\beta\ \cos\alpha - \cos\beta\ \sin\alpha$
$= (\frac{5}{13})\ (-\frac{3}{5}) - (-\frac{12}{13})\ (-\frac{4}{5}) = -\frac{63}{65}$

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3. Verify the identity $1 - \sec^4 x = -\dfrac{\sin^2 x\ (1 + \cos^2 x)}{\cos^4 x}$ (3 pts)

<u>Ans</u>: L.H.S. $= (1 - \sec^2 x)\ (1 + \sec^2 x) = (-\tan^2 x)\ (1 + \sec^2 x) = -\dfrac{\sin^2 x}{\cos^2 x} \cdot (1 + \dfrac{1}{\cos^2 x})$

$= -\dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x + 1}{\cos^2 x} = -\dfrac{\sin^2 x\ (1 + \cos^2 x)}{\cos^4 x} =$ R.H.S.

Math 002.16 **Quiz # 3** **Sem. II , 3 – 4 – 2005**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Find the domain and the range of the function $y = -2\sec(3x + \frac{\pi}{4}) + 5$ (3 pts)

<u>Ans</u>: For domain: $3x + \frac{\pi}{4} \neq (2n+1)\ \frac{\pi}{2} \implies 3x \neq n\pi + \frac{\pi}{2} - \frac{\pi}{4} = n\pi + \frac{\pi}{4} = (4n+1)\ \frac{\pi}{4} \implies x \neq (4n+1)\ \frac{\pi}{12}, n \in Z$. Therefore the domain $= (-\infty, \infty) - \{(4n+1)\ \frac{\pi}{12}, n \in Z\}$.

For range: $a = -2, d = 5$, the range $= (-\infty, -|a| + d] \cup [|a| + d, \infty) = (-\infty, -2+5] \cup [2+5, \infty)$ $= (-\infty, 3] \cup [7, \infty)$

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2. Find the vaues of the following: $\cos 165^\circ$, $\csc 15^\circ$, $\dfrac{\tan(\frac{\pi}{7} + \frac{2\pi}{3}) - \tan\frac{\pi}{7}}{1 + \tan(\frac{\pi}{7} + \frac{2\pi}{3}) \cdot \tan\frac{\pi}{7}}$ (4 pts)

<u>Ans</u>: (i) $\cos 165^\circ = \cos(45^\circ + 120^\circ) = \cos 45^\circ\ \cos 120^\circ - \sin 45^\circ\ \sin 120^\circ$ $= (\frac{\sqrt{2}}{2})\ (-\frac{1}{2}) - (\frac{\sqrt{2}}{2})\ (\frac{\sqrt{3}}{2}) = -\dfrac{\sqrt{2} + \sqrt{6}}{4}$

(ii) $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ\ \cos 30^\circ - \cos 45^\circ\ \sin 30^\circ = (\frac{\sqrt{2}}{2})\ (\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2})\ (\frac{1}{2})$

$= \dfrac{\sqrt{6} - \sqrt{2}}{4} \implies \csc 15^\circ = \dfrac{1}{\sin 15^\circ} = \dfrac{4}{\sqrt{6} - \sqrt{2}} \cdot \dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \dfrac{4\ (\sqrt{6} + \sqrt{2})}{6 - 2} = \sqrt{6} + \sqrt{2}$

(iii) $\dfrac{\tan(\frac{\pi}{7} + \frac{2\pi}{3}) - \tan\frac{\pi}{7}}{1 + \tan(\frac{\pi}{7} + \frac{2\pi}{3}) \cdot \tan\frac{\pi}{7}} = \tan[(\frac{\pi}{7} + \frac{2\pi}{3}) - \frac{\pi}{7}] = \tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

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3. Verify the identity $\dfrac{2\cos x}{\sin x - 1} + \dfrac{\cos x}{1 - \sin x} = -\dfrac{1 + \sin x}{\cos x}$ (3 pts)

<u>Ans</u>: L.H.S. $= -\dfrac{2\cos x}{1 - \sin x} + \dfrac{\cos x}{1 - \sin x} = -\dfrac{\cos x}{1 - \sin x} \cdot \dfrac{1 + \sin x}{1 + \sin x} = -\dfrac{\cos x\ (1 + \sin x)}{1 - \sin^2 x}$ $= -\dfrac{\cos x\ (1 + \sin x)}{\cos^2 x} = -\dfrac{1 + \sin x}{\cos x} =$ R.H.S.

Math 002.29 **Quiz # 3** **Sem. II , 3 – 4 – 2005**

Name : ________________________ **ID. # :** ______ **SER. # :** ____

1. Find the vertical asymptotes and the x–intercepts of the graph of $y = 4\cot(2x + \frac{\pi}{2})$ over the interval $[-\pi, \frac{3\pi}{2}]$. (no need to graph) (3 pts)

Ans: The graph has vertical asymptotes when $2x + \frac{\pi}{2} = n\,\pi \implies 2x = n\,\pi - \frac{\pi}{2} = (2n-1)\,\frac{\pi}{2}$ $\implies x = (2n-1)\,\frac{\pi}{4},\ n \in Z$. So over the interval $[-\pi, \frac{3\pi}{2}]$: if $n = -1$, $\boxed{x = -\frac{3\pi}{4}}$;

if $n = 0$, $\boxed{x = -\frac{\pi}{4}}$; if $n = 1$, $\boxed{x = \frac{\pi}{4}}$; if $n = 2$, $\boxed{x = \frac{3\pi}{4}}$; if $n = 3$, $\boxed{x = \frac{5\pi}{4}}$.

While the graph has x–intercepts when $2x + \frac{\pi}{2} = (2n+1)\frac{\pi}{2} \implies 2x = (2n)\,\frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2}$ $= n\,\pi \implies x = n\,\frac{\pi}{2},\ n \in Z$. So over the interval $[-\pi, \frac{3\pi}{2}]$: if $n = -2$, $\boxed{x = -\pi}$; if $n = -1$, $\boxed{x = -\frac{\pi}{2}}$; if $n = 0$, $\boxed{x = 0}$; if $n = 1$, $\boxed{x = \frac{\pi}{2}}$; if $n = 2$, $\boxed{x = \pi}$; if $n = 3$, $\boxed{x = \frac{3\pi}{2}}$

Therefore the x–intercepts are $(-\pi, 0)$, $(-\frac{\pi}{2}, 0)$, $(0, 0)$, $(\frac{\pi}{2}, 0)$, $(\pi, 0)$, and $(\frac{3\pi}{2}, 0)$.

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2. If $\sin\alpha = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$ and $\cos\beta = \frac{5}{13}$, $\frac{3\pi}{2} < \beta < 2\pi$, find

$\sin(\alpha + \frac{5\pi}{3})$, $\cos(\frac{\pi}{2} + \alpha - \beta)$, $\dfrac{\tan(\frac{\alpha}{2} + \beta) + \tan(\frac{\alpha}{2} - \beta)}{1 - \tan(\frac{\alpha}{2} + \beta) \cdot \tan(\frac{\alpha}{2} - \beta)}$ (4 pts)

Ans: α lies in Q II, $\sin\alpha = \frac{3}{5} = \frac{y}{r} \implies y = 3, r = 5, x = -\sqrt{25 - 9} = -4 \implies \cos\alpha = -\frac{4}{5}$, $\tan\alpha = -\frac{3}{4}$ and β lies in Q IV, $\cos\beta = \frac{5}{13} = \frac{x}{r} \implies x = 5, r = 13, y = -\sqrt{169 - 25} = -12$ $\implies \sin\beta = -\frac{12}{13}$. So

(i) $\sin(\alpha + \frac{5\pi}{3}) = \sin\alpha\ \cos\frac{5\pi}{3} + \cos\alpha\ \sin\frac{5\pi}{3} = (\frac{3}{5})(\frac{1}{2}) + (-\frac{4}{5})(-\frac{\sqrt{3}}{2}) = \frac{3 + 4\sqrt{3}}{10}$

(ii) $\cos(\frac{\pi}{2} + \alpha - \beta) = \cos[\frac{\pi}{2} - (\beta - \alpha)] = \sin(\beta - \alpha) = \sin\beta\ \cos\alpha - \cos\beta\ \sin\alpha$

$= (-\frac{12}{13})\,(-\frac{4}{5}) - (\frac{5}{13})\,(\frac{3}{5}) = \frac{48 - 15}{65} = \frac{33}{65}$

(iii) $\dfrac{\tan(\frac{\alpha}{2} + \beta) + \tan(\frac{\alpha}{2} - \beta)}{1 - \tan(\frac{\alpha}{2} + \beta) \cdot \tan(\frac{\alpha}{2} - \beta)} = \tan[(\frac{\alpha}{2} + \beta) + (\frac{\alpha}{2} - \beta)] = \tan\alpha = -\frac{3}{4}$

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3. Verify the identity $\sqrt{\dfrac{1 + \cos x}{1 - \cos x}} = \dfrac{1 + \cos x}{\sin(-x)}$, $\sin x < 0$ (3 pts)

Ans: L.H.S. $= \sqrt{\dfrac{1 + \cos x}{1 - \cos x} \cdot \dfrac{1 + \cos x}{1 + \cos x}} = \sqrt{\dfrac{(1 + \cos x)^2}{1 - \cos^2 x}} = \sqrt{\dfrac{(1 + \cos x)^2}{\sin^2 x}} = \dfrac{|1 + \cos x|}{|\sin x|}$

$= \dfrac{1 + \cos x}{-\sin x} = \dfrac{1 + \cos x}{\sin(-x)} =$ R.H.S. (because $1 + \cos x > 0$ and $\sin x < 0$ (given))