Math 002.08 **Quiz # 2** **Sem. II , 19 – 3 – 2005**

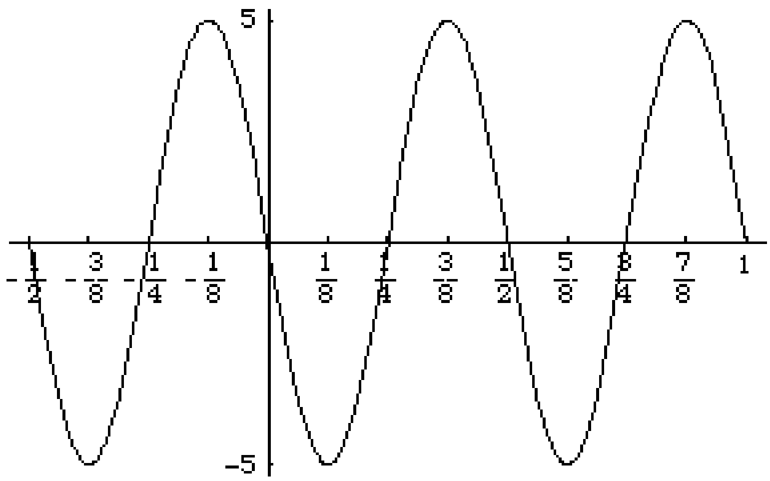
Name :________________________ **ID. # :** ______ **SER. # :** ____

- **Be clean and show your work**

1. Graph $y = -5\sin(4\pi x)$, $-\frac{1}{2} \leq x \leq 1$. Write down the period and the range. (5 pts)

Ans: $a = -5$, $b = 4\pi$, so amplitude $= |a| = 5$, range $= [-5, 5]$, and period $= \frac{2\pi}{b} = \frac{2\pi}{4\pi} = \frac{1}{2}$.

$\frac{1}{4} \cdot$ period $= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$



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2. (a) Find the value of: $\cos\left(-\frac{259\pi}{6}\right) + \tan\left(-\frac{49\pi}{3}\right)$ (3 pts)

Ans: $= \cos\left(\frac{259\pi}{6}\right) - \tan\left(\frac{49\pi}{3}\right) = \cos\left(42\pi + \frac{7\pi}{6}\right) - \tan\left(16\pi + \frac{\pi}{3}\right)$
$= \cos\frac{7\pi}{6} - \tan\frac{\pi}{3} = -\cos\frac{\pi}{6} - \tan\frac{\pi}{3} = -\frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{3\sqrt{3}}{2}$

(b) If $(-\frac{3}{5}, \frac{4}{5})$ is the point on the unit circle corresponding to an arc length t, find the point on the unit circle corresponding to (i) $5\pi - t$ (ii) $\frac{\pi}{2} - t$ (2 pts)

Ans: If (x, y) is the point on the unit circle corresponding to an arc length t , then the points on the unit circle corresponding to an arc lengths $-t$, $5\pi - t$, $\frac{\pi}{2} - t$ respectively are $(x, -y)$, $(-x, y)$, (y, x). Therefore

$t : (-\frac{3}{5}, \frac{4}{5})$

$5\pi - t : (\frac{3}{5}, \frac{4}{5})$

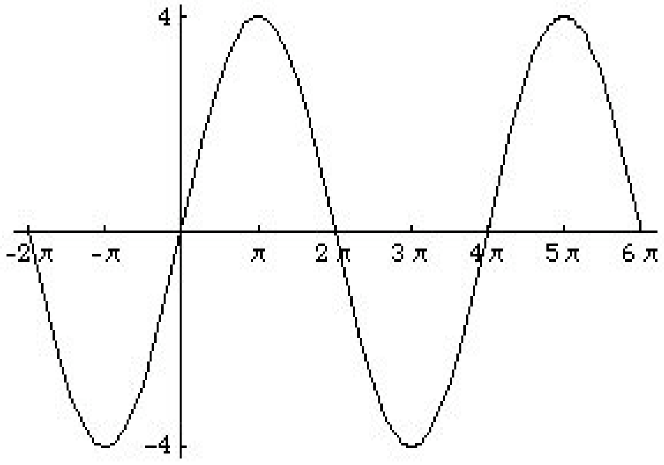
$\frac{\pi}{2} - t : (\frac{4}{5}, -\frac{3}{5})$

Math 002.16 **Quiz # 2** **Sem. II , 19 – 3 – 2005**

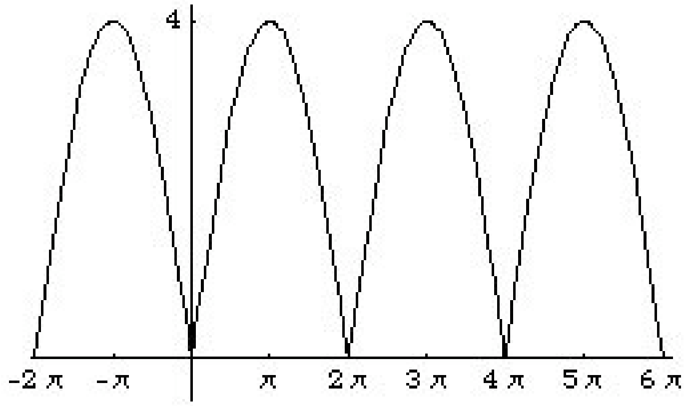
Name :____________________________ **ID. # :** ______ **SER. # :** ____

1. Graph $y = -4\left|\sin \frac{1}{2}x\right|$, $-2\pi \le x \le 6\pi$. Write down the period and the range. (5 pts)

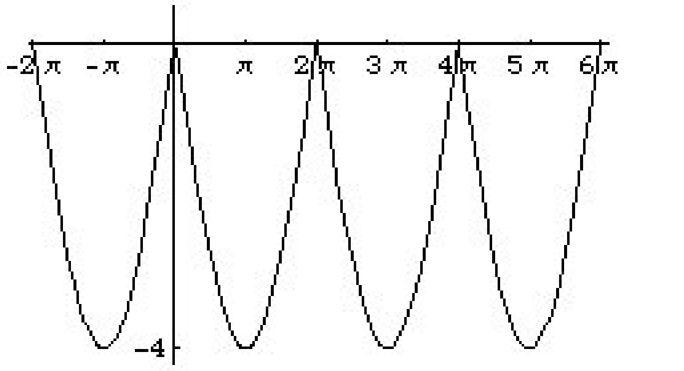
Ans: First, we draw $y = 4\sin\frac{1}{2}x$, where $a = 4$, $b = \frac{1}{2}$, so amplitude $= |a| = 4$, period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$, $\frac{1}{4}\cdot$ period $= \frac{1}{4}\cdot 4\pi = \pi$. Next, we draw $y = 4\left|\sin\frac{1}{2}x\right|$ by keeping the positive parts of the first graph and reflecting the negative parts of it across the x–axix. Finally, we reflect the latest graph across the x–axis to get $y = -4\left|\sin\frac{1}{2}x\right|$. Clearly from the third graph, the range $= [-4, 0]$, and the period $= 2\pi$.



y = 4 sin x/2



y = 4 l sin x/2 l



y = - 4 l sin x/2 l

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2. (a) Find $W\left(-\frac{451\pi}{6}\right)$, where $W(t)$ is the wrapping function. (3 pts)

Ans: $= \left(\cos(-\frac{451\pi}{6}), \sin(-\frac{451\pi}{6})\right) = \left(\cos\frac{451\pi}{6}, -\sin\frac{451\pi}{6}\right)$

$= \left(\cos(74\pi + \frac{7\pi}{6}), -\sin(74\pi + \frac{7\pi}{6})\right) = \left(\cos\frac{7\pi}{6}, -\sin\frac{7\pi}{6}\right) = \left(-\cos\frac{\pi}{6}, \sin\frac{\pi}{6}\right)\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(b) Is the function $f(x) = \dfrac{\tan^2 x \ + \ \sec 3x}{4x^3}$ even, odd, or neither? (explain) (2 pts)

Ans: $f(-x) = \dfrac{\left(\tan(-x)\right)^2 + \sec(-3x)}{4(-x)^3} = \dfrac{(-\tan x)^2 + \sec 3x}{-4x^3} = \dfrac{\tan^2 x + \sec 3x}{-4x^3} = -\dfrac{\tan^2 x + \sec 3x}{4x^3} = -f(x)$, so $f(x)$ is an odd function.

Math 002.29 **Quiz # 2** **Sem. II , 19 – 3 – 2005**

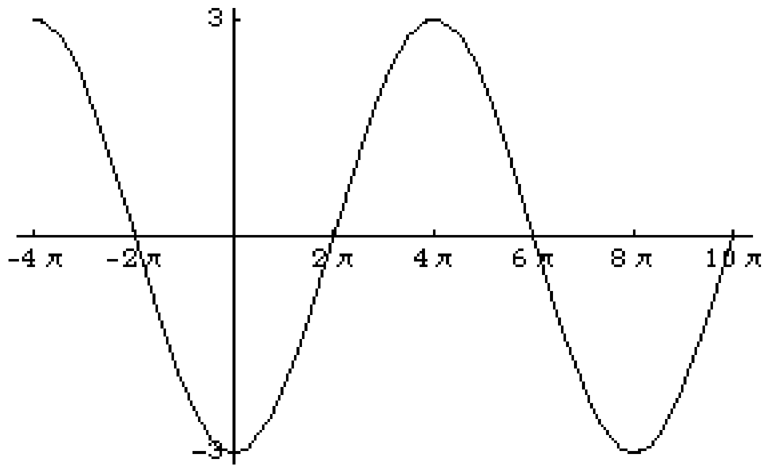
Name :______________________________ **ID. # :** ______ **SER. # :** ____

- **Be clean and show your work**

1. Graph $y = -3\cos\frac{1}{4}x$, $-4\pi \le x \le 10\pi$. Write down the period and the range. (5 pts)

Ans: $a = -3$, $b = \frac{1}{4}$, so amplitude $= |a| = 3$, range $= [-3, 3]$, and period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{4}} = 8\pi$.

$\frac{1}{4} \cdot$ period $= \frac{1}{4} \cdot 8\pi = 2\pi$.



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2. (a) Find the value of: $\sec\left(-\frac{161\pi}{3}\right) + \cot\left(-\frac{97\pi}{4}\right)$ (3 pts)

Ans: $= \sec\left(\frac{161\pi}{3}\right) - \cot\left(\frac{97\pi}{4}\right) = \sec\left(52\pi + \frac{5\pi}{3}\right) - \cot\left(24\pi + \frac{\pi}{4}\right)$
$= \sec\frac{5\pi}{3} - \cot\frac{\pi}{3} = \sec\frac{\pi}{3} - \cot\frac{\pi}{4} = 2 - 1 = 1$

(b) If $\pi < x < \frac{3\pi}{2}$, write $\tan x$ in terms of $\sin x$ (only). (2 pts)

Ans: x lies in quadrant III.

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sin x}{-\sqrt{1 - \sin^2 x}} = -\frac{\sin x}{\sqrt{1 - \sin^2 x}}$$