Math 002.07 **Quiz # 3** **Sem. II , 17 – 4 – 2006**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Find the value of: (i) $\cos^2 112.5^\circ - \frac{1}{2}$ (ii) $\sin\left[\frac{1}{2}\tan^{-1}(-\frac{3}{4})\right]$ (3.5 pts)

Solution: (i) $= \frac{1}{2}(2\cos^2 112.5^\circ - 1) = \frac{1}{2}\cos 2(112.5^\circ) = \frac{1}{2}\cos 225^\circ = \frac{1}{2}(-\cos 45^\circ) = -\frac{1}{2}\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{4}$

(ii) Let $\alpha = \tan^{-1}(-\frac{3}{4}) \in$ Q IV$^-$ $\implies$ $\tan\alpha = -\frac{3}{4}$ $(y = -3\ ,\ x = 4,\ \text{so}\, r = 5)$ $\implies$ $\cos\alpha = \frac{4}{5}$

Now $-\frac{\pi}{2} < \alpha < 0 \implies -\frac{\pi}{4} < \frac{\alpha}{2} < 0 \implies \frac{\alpha}{2} \in$ Q IV$^-$

Therefore $\sin\left[\frac{1}{2}\tan^{-1}(-\frac{3}{4})\right] = \sin\frac{\alpha}{2} = -\sqrt{\frac{1-\cos\alpha}{2}} = -\sqrt{\frac{1-\frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = -\frac{\sqrt{10}}{10}$

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2. Find the **phase shift** and the **range** of the graph of the function
$y = (-3\sqrt{3}\sin 2x + 3\cos 2x) + 1$ (3.5 pts)

Solution: $\mathrm{A} = -3\sqrt{3}$, $\mathrm{B} = 3$, $\mathrm{b} = 2$, $\mathrm{d} = 1$ $\implies$ $k = \sqrt{\mathrm{A}^2 + \mathrm{B}^2} = \sqrt{27+9} = 6$, $\cos\alpha = \frac{\mathrm{A}}{k}$
$= -\frac{3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$, $\sin\alpha = \frac{\mathrm{B}}{k} = \frac{3}{6} = \frac{1}{2}$ $\implies$ $\alpha \in$ Q II , $\alpha' = \frac{\pi}{6}$ $\implies$ $\alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Then $y = 6\sin(2x + \frac{5\pi}{6}) + 1$ $\implies$ phase shift $= \frac{-\alpha}{b} = \frac{-\frac{5\pi}{6}}{2} = -\frac{5\pi}{12}$

and range $= [-k+d, k+d] = [-6+1, 6+1] = [-5, 7]$

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3. Find the exact value of $\sin^{-1}(-\frac{4}{5}) + \cos^{-1}\frac{3}{5}$ (show your work) (3 pts)

Solution: $\sin^{-1}(-\frac{4}{5}) + \cos^{-1}\frac{3}{5} = -\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{3}{5}$. If we call $\alpha = \cos^{-1}\frac{3}{5} \in$ Q I,

then $\cos\alpha = \frac{3}{5}$ $(x = 3, r = 5, \text{so}\, y = 4)$ $\implies$ $\sin\alpha = \frac{4}{5}$ $\implies$ $\alpha = \sin^{-1}\frac{4}{5} = \cos^{-1}\frac{3}{5}$.

Therefore the value $= -\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{3}{5} = -\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5} = 0$

Math 002.23 **Quiz # 3** **Sem. II , 17 – 4 – 2006**

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1. Find the value of: (i) $\dfrac{2\tan 75^\circ}{1+\tan^2 75^\circ}$ (ii) $\sin\left[\frac{1}{2}\csc^{-1}(-\frac{5}{3})\right]$ (3.5 pts)

Solution: (i) $= \dfrac{\frac{2\sin 75^\circ}{\cos 75^\circ}}{\sec^2 75^\circ} = \dfrac{\frac{2\sin 75^\circ}{\cos 75^\circ}}{\frac{1}{\cos^2 75^\circ}} = 2\sin 75^\circ \cos 75^\circ = \sin 2(75^\circ) = \sin 150^\circ = \sin 30^\circ = \frac{1}{2}$

(ii) Let $\alpha = \csc^{-1}(-\frac{5}{3}) \in$ Q IV^- $\implies$ $\csc\alpha = -\frac{5}{3}$ $(r = 5\ ,\ y = -3,\ \text{so}\ x = 4)$ $\implies$ $\cos\alpha = \frac{4}{5}$

Now $-\frac{\pi}{2} < \alpha < 0 \implies -\frac{\pi}{4} < \frac{\alpha}{2} < 0 \implies \frac{\alpha}{2} \in$ Q IV^-

Therefore $\sin\left[\frac{1}{2}\csc^{-1}(-\frac{5}{3})\right] = \sin\frac{\alpha}{2} = -\sqrt{\frac{1-\cos\alpha}{2}} = -\sqrt{\frac{1-\frac{4}{5}}{2}} = -\sqrt{\frac{1}{10}} = -\frac{\sqrt{10}}{10}$

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2. Find the **phase shift** and the **range** of the graph of the function
$y = (-6\sin\frac{x}{2} - 6\cos\frac{x}{2}) + 3$ (3.5 pts)

Solution: A $= -6$, B $= 6$, b $= \frac{1}{2}$ d $= 3$ $\implies$ $k = \sqrt{\text{A}^2+\text{B}^2} = \sqrt{36+36} = 6\sqrt{2}$, $\cos\alpha = \frac{\text{A}}{k}$
$= -\frac{-6}{6\sqrt{2}} = -\frac{\sqrt{1}}{\sqrt{2}}$, $\sin\alpha = \frac{\text{B}}{k} = \frac{-6}{6\sqrt{2}} = -\frac{\sqrt{1}}{\sqrt{2}}$ $\implies$ $\alpha \in$ Q III , $\alpha' = \frac{\pi}{4}$ $\implies$ $\alpha = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

Then $y = 6\sqrt{2}\sin(\frac{x}{2}+\frac{5\pi}{4}) + 3$ $\implies$ phase shift $= \frac{-\alpha}{b} = \dfrac{-\frac{5\pi}{4}}{\frac{1}{2}} = -\frac{5\pi}{2}$

and range $= [-k+d, k+d] = [-6\sqrt{2}+3, 6\sqrt{2}+3]$

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3. Find the exact value of $\cos^{-1}(-\frac{5}{13}) + \sin^{-1}\frac{12}{13}$ (show your work) (3 pts)

Solution: $\cos^{-1}(-\frac{5}{13}) + \sin^{-1}\frac{12}{13} = \left(\pi - \cos^{-1}\frac{5}{13}\right) + \sin^{-1}\frac{12}{13}$.

If we call $\alpha = \cos^{-1}(\frac{5}{13}) \in$ Q I, then $\cos\alpha = \frac{5}{13}$ $(x = 5, r = 13, \text{so}\ y = 12)$ $\implies$ $\sin\alpha = \frac{12}{13}$
$\implies$ $\alpha = \sin^{-1}\frac{12}{13} = \cos^{-1}\frac{5}{13}$.

Therefore the value $= \left(\pi - \sin^{-1}\frac{12}{13}\right) + \sin^{-1}\frac{12}{13} = \pi$

Math 002.30 **Quiz # 3** **Sem. II , 17 – 4 – 2006**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Find the value of: (i) $-\sin 67.5^\circ \cos 67.5^\circ$ (ii) $\cos\left[\frac{1}{2}\cot^{-1}\frac{7}{24}\right]$ (3.5 pts)

<u>**Solution**</u>: (i) $-\frac{1}{2}(2\sin 67.5^\circ \cos 67.5^\circ) = -\frac{1}{2}\sin 2(67.5^\circ) = -\frac{1}{2}\sin 135^\circ = -\frac{1}{2}(\sin 45^\circ) = -\frac{\sqrt{2}}{4}$

(ii) Let $\alpha = \cot^{-1}\frac{7}{24} \in$ Q I $\implies \cot\alpha = \frac{7}{24}$ $(x = 7\ ,\ y = 24,\ \text{so}\, r = 25)$ $\implies \cos\alpha = \frac{7}{25}$

Now $0 < \alpha < \frac{\pi}{2} \implies 0 < \frac{\alpha}{2} < \frac{\pi}{4} \implies \frac{\alpha}{2} \in$ Q I

Therefore $\cos\left[\frac{1}{2}\cot^{-1}\frac{7}{24}\right] = \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

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2. Find the **amplitude**, **period**, and **phase shift** of the graph of the function $y = (-4\sin\frac{x}{3} + 4\sqrt{3}\cos\frac{x}{3}) - 5$ (3.5 pts)

<u>**Solution**</u>: A $= -4$, B $= 4\sqrt{3}$, b $= \frac{1}{3}$ $\implies k = \sqrt{\text{A}^2 + \text{B}^2} = \sqrt{16+48} = \sqrt{64} = 8$, $\cos\alpha = \frac{\text{A}}{k}$ $= -\frac{4}{8} = -\frac{1}{2}$, $\sin\alpha = \frac{\text{B}}{k} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \implies \alpha \in \text{Q II}\,, \alpha' = \frac{\pi}{3} \implies \alpha = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

Then $y = 8\sin(2x + \frac{2\pi}{3}) - 5 \implies$ amplitude = k = 8, period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 6\pi$,

and phase shift = $\frac{-\alpha}{b} = \frac{-\frac{2\pi}{3}}{\frac{1}{3}} = -2\pi$

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3. Find the **domain** and the **range** of the function $y = -2\csc^{-1} 2x + 3\pi$ (3 pts)

<u>**Solution**</u>: For domain: $2x \in$ domain of $\csc^{-1} x = (-\infty, -1] \cup [1, \infty) \implies 2x \le -1$ or $2x \ge 1$ $\implies x \le -\frac{1}{2}$ or $x \ge \frac{1}{2} \implies$ D $= (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$.

$\csc^{-1} 2x \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\} \implies -2\csc^{-1} 2x \in [-\pi, \pi] - \{0\} \implies -2\csc^{-1} 2x + 3\pi \in [2\pi, 4\pi] - \{3\pi\}$ $\implies y \in [2\pi, 4\pi] - \{3\pi\} \implies$ the range $= [2\pi, 3\pi) \cup (3\pi, 4\pi]$