Math 002.07 **Quiz # 1** **Sem. II , 26 – 2 – 2006**

Name :____________________ **ID. # :** ______ **SER. # :** _____

1. (a) Find the domain and asymptote(s) of $y = \log_2(x^2 - x - 2)$ (2 pts)

Solution: $x^2 - x - 2 > 0 \implies (x+1)(x-2) > 0$, by sign test:

$(x+1)$	$----$	$++++$	$++++$
$(x-2)$	$----$	$----$	$++++$
		-1	2
L.S. :	$(+)$	$(-)$	$(+)$

Therefore the domain $= (-\infty, -1) \cup (2, \infty)$.

The vertical asymptotes are $\boxed{x = -1}$ and $\boxed{x = 2}$

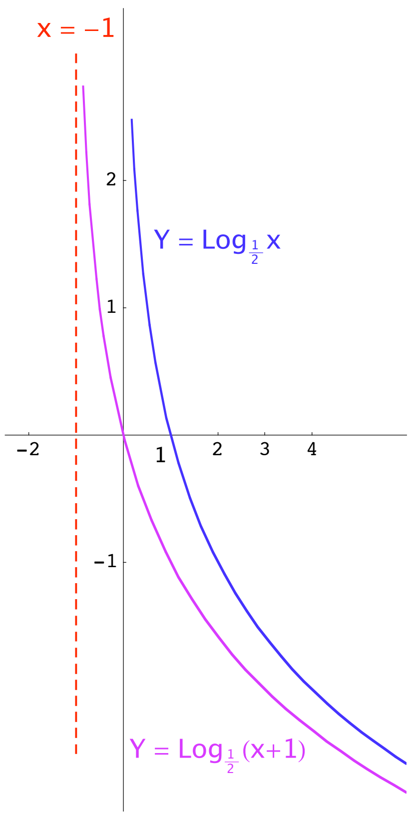
(b) Find the range and asymptote(s) of $y = 10^{(1-3x)} - \frac{1}{2}$ (2 pts)

Solution: We know that $10^{(1-3x)} > 0 \implies 10^{(1-3x)} - \frac{1}{2} > -\frac{1}{2} \implies y > -\frac{1}{2}$,

so the range $= (-\frac{1}{2}, \infty)$. The horizontal asymptote is $\boxed{y = -\frac{1}{2}}$

(c) Graph $y = \log_{\frac{1}{2}}(x+1)$ (show your work) (2 pts)

Solution: First, we draw $y = \log_{\frac{1}{2}} x$ which is decreasing and has x–intercept $= (1, 0)$, v. asymptote $x = 0$. Then translate this graph 1 unit to left to get $y = \log_{\frac{1}{2}}(x+1)$ with x–intercept $= (0, 0)$, v. asymptote $x = -1$.



2. (a) Find the value of: $\log(\ln e^{100})$, $e^{(2-3\ln 2)}$, $(9)^{\left(\frac{\log 5}{\log 3}\right)}$ (2 pts)

Solution:

(i) $\log(\ln e^{100}) = \log(100 \ln e) = \log 100 = \log 10^2 = 2$

(ii) $e^{(2-3\ln 2)} = e^2 \cdot e^{-3\ln 2} = e^2 \cdot 2^{-3} = \frac{e^2}{8}$

(iii) $(9)^{\left(\frac{\log 5}{\log 3}\right)} = (3)^{\left(\frac{2\log 5}{\log 3}\right)} = (3)^{\left(\frac{\log 25}{\log 3}\right)} = (3)^{\log_3 25} = 25$

(b) Write as a single logarithm: $3\log_3(xy) + 2\log_3\sqrt{yz} - 2\log_3 4 - \log_{\sqrt{3}}(x^2 z)$,
where $x, y, z > 0$ (2 pts)

Solution: $\log_3(x^3y^3)+\log_3(yz)-(\log_3 16+\frac{\log_3(x^2z)}{\log_3\sqrt{3}}) = \log_3(x^3y^4z)-\left(\log_3 16+\frac{\log_3(x^2z)}{\frac{1}{2}}\right)$

$= \log_3(x^3y^4z) - [\log_3 16 + 2\log_3(x^2z)] = \log_3(x^3y^4z) - \log_3(16x^4z^2) = \log_3\left(\frac{x^3y^4z}{16x^4z^2}\right)$

$= \log_3\left(\frac{y^4}{16xz}\right)$

Math 002.23 **Quiz # 1** **Sem. II , 26 − 2 − 2006**

Name :________________________ **ID. # :** ______ **SER. # :** ____

1. (a) Find the domain and asymptote(s) of $y = \log_{\frac{1}{3}}(x^2 - 5x - 6)$ (2 pts)

<u>Solution</u>: $x^2 - 5x - 6 > 0 \implies (x+1)(x-6) > 0$, by sign test:

		-1	6
$(x+1)$	$- - - -$	$+ + + +$	$+ + + +$
$(x-6)$	$- - - -$	$- - - -$	$+ + + +$
L.S. :	$(+)$	$(-)$	$(+)$

Therefore the domain $= (-\infty, -1) \cup (6, \infty)$.

The vertical asymptotes are $\boxed{x = -1}$ and $\boxed{x = 6}$

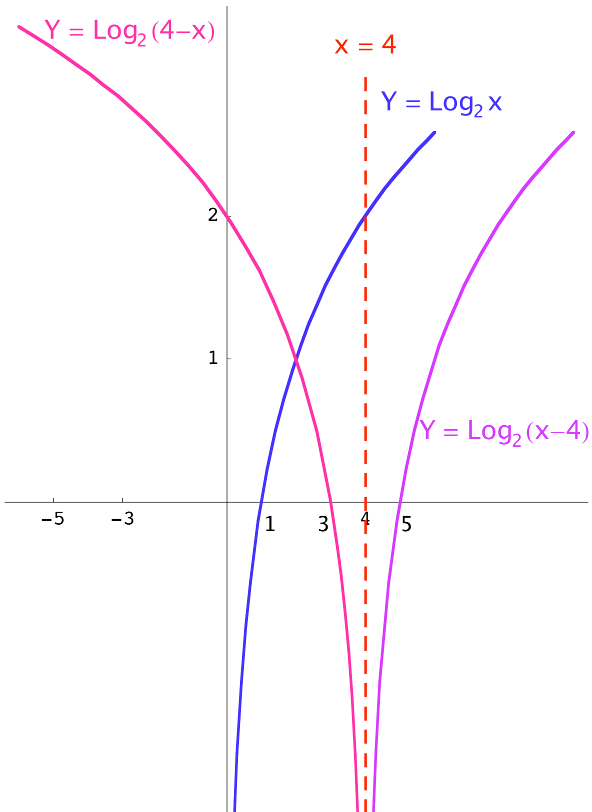
(b) Find the range and asymptote(s) of $y = 3 - e^{(x+2)}$ (2 pts)

<u>Solution</u>: We know that $e^{(x+2)} > 0 \implies -e^{(x+2)} < 0 \implies 3 - e^{(x+2)} < 3 \implies y < 3$,

so the range $= (-\infty, 3)$. The horizental asymptote is $\boxed{y = 3}$

(c) Graph $y = \log_2(4 - x)$ (show your work) (2 pts)

<u>Solution</u>: First, we draw $y = \log_2 x$ which is increasing and has x–intercept $= (1, 0)$, v. asymptote $x = 0$. Then translate this graph 4 unit to right to get $y = \log_2(x - 4)$ with x–intercept $= (4, 0)$, v. asymptote $x = 4$. Finally reflect the last graph across the line $x = 4$ to get $y = \log_2(4 - x)$ with x–intercept $= (3, 0)$, y–intercept $= (0, \log_2 4) = (0, 2)$, and v. asymptote $x = 4$.



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2. (a) Find the value of: $\ln\left[\log_{\frac{1}{2}} 2^{(-e^5)}\right]$, $16^{(1+\frac{1}{2}\log_2 3)}$, $(e)^{\left(\frac{2\log 3}{\log e}\right)}$ (2 pts)

Solution:

(i) $\ln\left[\log_{\frac{1}{2}} 2^{(-e^5)}\right] = \ln[\log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{e^5}] = \ln(e^5) = 5$

(ii) $16^{(1+\frac{1}{2}\log_2 3)} = 16 \cdot \left(2^4\right)^{(\frac{1}{2}\log_2 3)} = 16 \cdot (2)^{(2\log_2 3)} = 16 \cdot (2)^{(\log_2 9)} = (16)(9) = 144$

(iii) $(e)^{\left(\frac{2\log 3}{\log e}\right)} = (e)^{\left(\frac{\log 9}{\log e}\right)} = e^{\ln 9} = 9$

(b) If $f(x) = -2 + \log_2(5-x)$, find $f^{-1}(x)$ (2 pts)

Solution: f is a 1–1 function, so it has an inverse. To find it, interchange x with y:

$x = -2 + \log_2(5-y)$, next solve for y

$x + 2 = \log_2(5-y) \implies 5 - y = 2^{(x+2)} \implies y = f^{-1}(x) = 5 - 2^{(x+2)}$

Math 002.30 **Quiz # 1** **Sem. II , 26 – 2 – 2006**

Name : ______________________________ **ID. # :** ______ **SER. # :** ____

1. (a) Find the domain and asymptote(s) of $y = \log(x^2 - 4)$ (2 pts)

<u>**Solution**</u>: $x^2 - 4 > 0 \implies x^2 > 4 \implies |x| > 2 \implies x < -2$ or $x > 2$

$\implies$ Domain $= (-\infty, -2) \cup (2, \infty)$. The vertical asymptotes are $\boxed{x = -2}$ and $\boxed{x = 2}$

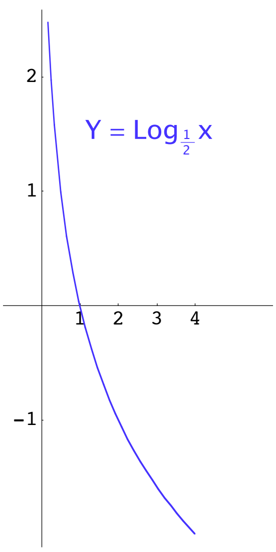
(b) Find the range and asymptote(s) of $y = 3 - e^{(x+2)}$ (2 pts)

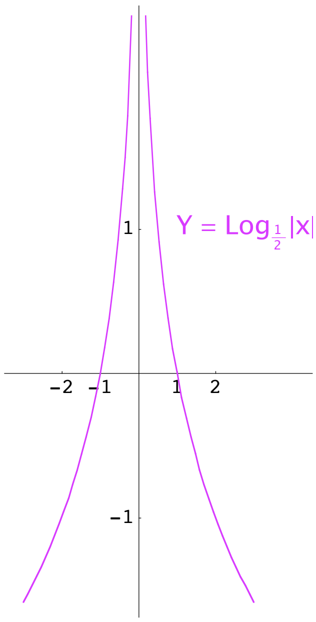
<u>**Solution**</u>: We know that $e^{(x+2)} > 0 \implies -e^{(x+2)} < 0 \implies 3 - e^{(x+2)} < 3 \implies y < 3$,

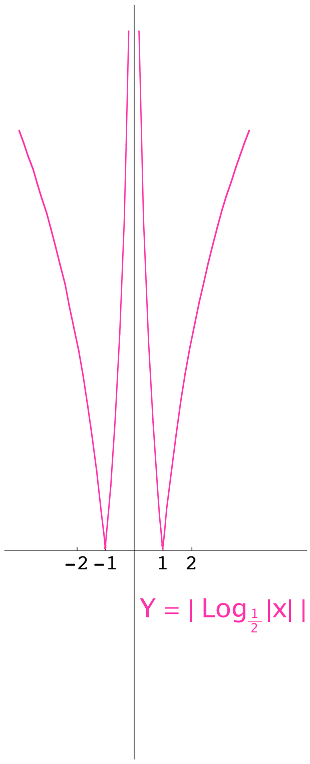
so the range $= (-\infty, 3)$. The horizental asymptote is $\boxed{y = 3}$

(c) Graph $y = \left|\log_{\frac{1}{2}} |x|\right|$ (show your work) (2 pts)

<u>**Solution**</u>: Add to the graph $y = \log_{\frac{1}{2}} x$ its reflection across y–axix to get the graph of $y = \log_{\frac{1}{2}} |x|$. Then keep the positive parts and reflect the negative parts of $y = \log_{\frac{1}{2}} |x|$ across the x–axis to get $y = \left|\log_{\frac{1}{2}} |x|\right|$. The x-intercepts $= (-1, 0)$, $(1, 0)$ and v. asymptote is $\boxed{x = 0}$







2. (a) Find the value of: $\log 20 + \log 300 - \log 6$, $\left(\frac{1}{2}\right)^{(2-3\log_2 5)}$, $(100)^{\left(\frac{2\ln 3}{\ln 10}\right)}$ (2 pts)

Solution:

(i) $\log 20 + \log 300 - \log 6 = \log(20)(300) - \log 6 = \log \dfrac{6000}{6} = \log 1000 = \log 10^3 = 3$

(ii) $\left(\frac{1}{2}\right)^{(2-3\log_2 5)} = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{-3\log_2 5} = \frac{1}{4} \cdot 2^{(3\log_2 5)} = \frac{1}{4} \cdot 2^{(\log_2 125)} = \frac{1}{4} \cdot (125) = \dfrac{125}{4}$

(iii) $(100)^{\left(\frac{2\ln 3}{\ln 10}\right)} = (100)^{\left(\frac{\ln 9}{\ln 10}\right)} = \left(10^2\right)^{\log 9} = 10^{2\log 9} = 10^{\log 81} = 81$

(b) If $f(x) = \left(\frac{1}{3}\right)^{(2x+5)} - 26$, find $f^{-1}(1)$ (2 pts)

Solution: $x = 1$ in the equation of $y = f^{-1}(x)$ is $y = 1$ in the equation of $y = f(x)$.

Thus $1 = \left(\frac{1}{3}\right)^{(2x+5)} - 26 \implies 27 = \left(\frac{1}{3}\right)^{(2x+5)} \implies (3)^{-(2x+5)} = 3^3$

$\implies -2x - 5 = 3 \implies 2x = -8 \implies x = -4 \implies f^{-1}(1) = -4$.