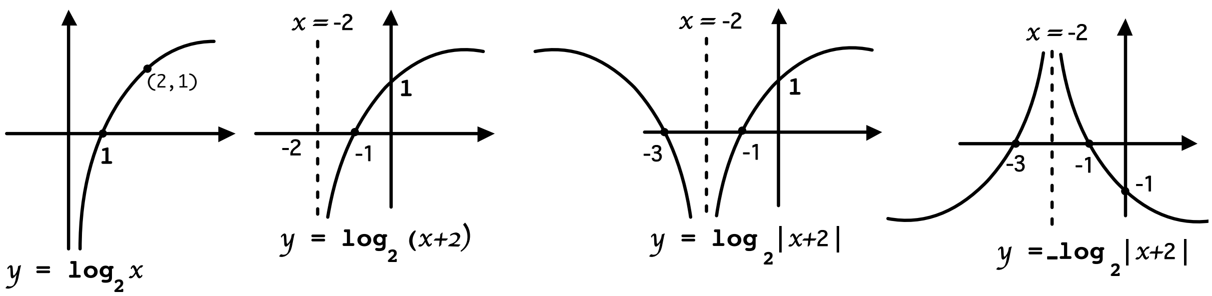
Math 002.07 **Quiz # 1** **Sem. II , 2 – 3 – 2008**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Graph $y = -\log_2 |x+2|$ showing all intercepts. Write down the domain and asymptote(s). (4 pts)

Ans: Shift the graph of $y = \log_2 x$ two units to the left to get $y = \log_2(x+2)$. Next add to the graph of $y = \log_2(x+2)$ its reflection across its symptote $x = -2$ to get $y = \log_2 |x+2|$. Finally reflect the last graph across the x–axis to form $y = -\log_2 |x+2|$.



Clearly the domain $= \Re - \{-2\} = (-\infty, -2) \cup (-2, \infty)$ and the vertical asymptote is $x = -2$

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2. If $\log 2 = m$, $\log 3 = n$, then find the value of: (3 pts)

$\log 0.008$, $\log 500$, $\log \dfrac{1}{\sqrt[3]{6}}$, $(10)^{(2\log 3 + 3\log 2)}$

Ans: $\log 0.008 = \log \dfrac{8}{1000} = \log 8 - \log 10^3 = 3\log 2 - 3 = 3m - 3 = 3(m-1)$

$\log 500 = \log 5 + \log 100 = \log \dfrac{10}{2} + 2 = \log 10 - \log 2 + 2 = 1 - m + 2 = 3 - m$

$\log \dfrac{1}{\sqrt[3]{6}} = \log 6^{-1/3} = -\dfrac{1}{3}\log 6 = -\dfrac{1}{3}(\log 2 + \log 3) = -\dfrac{1}{3}(m+n)$

$(10)^{(2\log 3 + 3\log 2)} = 10^{(\log 9 + \log 8)} = 10^{\log 72} = 72$

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3. Write as a single logarithm (assume all variables are positive): (3 pts)

$2\log_4(xy^2) + \dfrac{1}{2}\log_4(y^2 z^3) - \log_2 x + 2$

Ans: $= \log_4(x^2\, y^4) + \log_4(y\, z^{3/2}) - \dfrac{\log_4 x}{\log_4 2} + 2\log_4 4 = \log_4(x^2\, y^5\, z^{3/2}) - \dfrac{\log_4 x}{1/2} + \log_4 16$

$= \log_4(16\, x^2\, y^5\, z^{3/2}) - 2\log_4 x = \log_4\left(\dfrac{16\, x^2\, y^5\, z^{3/2}}{x^2}\right) = \log_4(16\, y^5\, z^{3/2})$

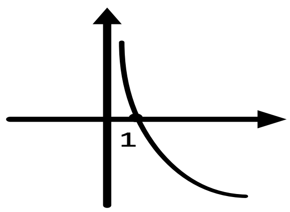
Be informed that $\log_4 x + 2 \neq \log_4(x+2)$

Math 002.15 **Quiz # 1** **Sem. II , 2 – 3 – 2008**

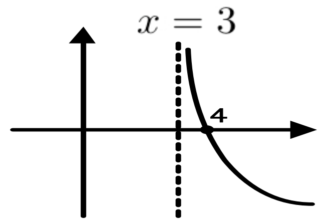
Name :____________________________ **ID. # :** ______ **SER. # :** ____

1. Graph $f(x) = -\log_{\frac{1}{3}}(3-x) + 1$ showing all intercepts. Is $f(x)$ increasing or decreasing? and where? (4 pts)

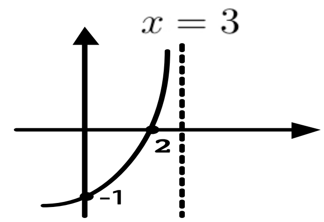
Ans: Shift the graph of $y = \log_{\frac{1}{3}} x$ three units to the right to get $y = \log_{\frac{1}{3}}(x-3)$, then reflect the new graph across its symptote $x = 3$ to have $y = \log_{\frac{1}{3}}(3-x)$. Next reflect the last one across the x–axis to form $y = -\log_{\frac{1}{3}}(3-x)$. Finally graph $-\log_{\frac{1}{3}}(3-x) + 1$ by shifting the last graph 1 unit upward.



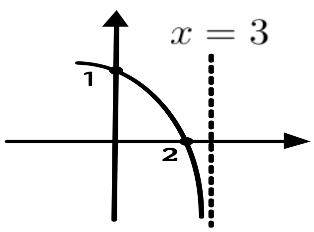
$y = \log_{\frac{1}{3}} x$



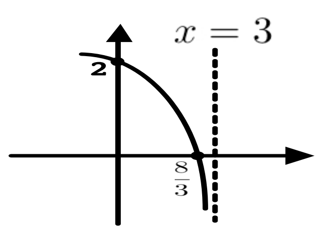
$y = \log_{\frac{1}{3}}(x-3)$



$y = \log_{\frac{1}{3}}(3-x)$



$y = -\log_{\frac{1}{3}}(3-x)$



$y = -\log_{\frac{1}{3}}(3-x) + 1$

$f(x)$ is decreasing on $(-\infty, 3)$

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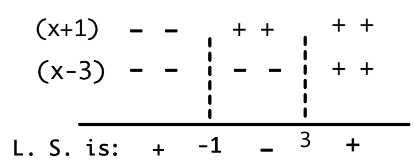
2. Find the domain of $f(x) = \log(x^2 - 2x - 3) + 1$ and the range of $g(x) = -\,2^{(x+3)} + 5$ (3 pts)

Ans: (i) $x^2 - 2x - 3 > 0 \implies (x+1)(x-3) > 0$
$\implies$ D $= (-\infty, -1) \cup (3, \infty)$



(ii) $2^{(x+3)} > 0 \implies -\,2^{(x+3)} < 0 \implies -\,2^{(x+3)} + 5 < 5 \implies y < 5 \implies$ R $= (-\infty, 5)$

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3. Write as a single logarithm (assume all variables are positive): (3 pts)

$$-2\ln(x\sqrt{y}) + \frac{1}{2}\ln(y^2 z^4) - \log_{\frac{1}{e}} z - 1$$

Ans: $= \ln(y\ z^2) - \ln(x^2\ y) - \dfrac{\ln z}{\ln(1/e)} - \ln e = \ln\left(\dfrac{y\ z^2}{x^2\ y}\right) - \dfrac{\ln z}{-1} - \ln e$

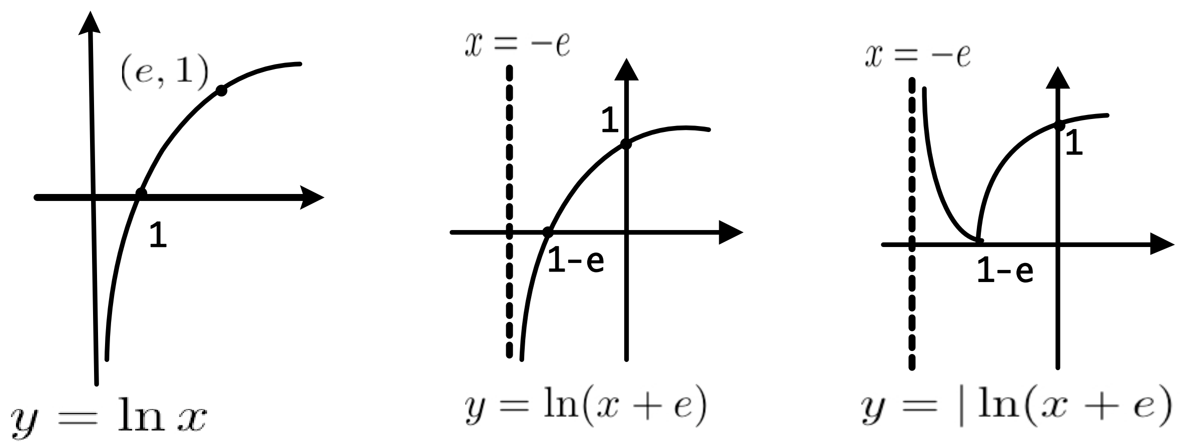
$$= \ln\left(\frac{z^2}{x^2}\right) + \ln z - \ln e = \ln\left(\frac{z^2}{x^2}\right) + \ln\left(\frac{z}{e}\right) = \ln\left(\frac{z^3}{e\ x^2}\right)$$

Math 002.26 **Quiz # 1** **Sem. II , 2 – 3 – 2008**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Graph $f(x) = \left|\ln(x+e)\right|$ showing all intercepts. Write down the domain and asymptote(s). (4 pts)

<u>Ans:</u> Shift the graph of $y = \ln x$, e units to the left to get $y = \ln(x+e)$ with an x–intercept $= (1-e, 0)$ and y-intercept $= (0, 1)$. Keep the positive part of this graph and reflect the negative part across the x–axis to have $y = |\ln(x+e)|$.



The domain $= (-e, \infty)$, range $= [0, \infty)$, and the vertical asymptote is $x = -e$

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2. If $f(x) = 2\log(x+4) - 1$, then find $f^{-1}(x)$. Also find the value of $f(-3.9)$, $f^{-1}(-1)$ (3 pts)

<u>Ans:</u> Interchange x with y: $x = 2\log(y+4) - 1$. Next solve for y: $x + 1 = 2\log(y+4)$

$$\Longrightarrow \log(y+4) = \frac{x+1}{2} \Longrightarrow y + 4 = (10)^{\frac{x+1}{2}} \Longrightarrow y = f^{-1}(x) = -4 + (10)^{\frac{x+1}{2}}.$$

$f(-3.9) = 2\log(-3.9+4) - 1 = 2\log(0.1) - 1 = 2(-1) - 1 = -3$ and $f^{-1}(-1) = -4 + (10)^0 = -4 + 1 = -3$

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3. Write as a single logarithm (assume all variables are positive): (3 pts)

$$\frac{1}{2}\log_3(y^2 z) + 2\log_3(xy^3) + \log_{\frac{1}{3}} z - 2$$

<u>Ans:</u> $= \log_3(y\ \sqrt{z}) + \log_3(x^2\ y^6) + \dfrac{\log_3 z}{\log_3 \frac{1}{3}} - 2\log_3 3 = \log_3(x^2\ y^7\ \sqrt{z}) - \log_3 z - \log_3 9$

$$= \log_3(x^2\ y^7\ \sqrt{z}) - (\log_3 z + \log_3 9) = \log_3(x^2\ y^7\ \sqrt{z}) - \log_3(9\ z) = \log_3\left(\frac{x^2\ y^7\ \sqrt{z}}{9\ z}\right)$$