Questions from old Exams

1 Section 4.2

1. Sketch the graph of $g(x) = -\left(\frac{1}{3}\right)^x + 3$. Write down the range of g and all asymptotes (if any).

2. Given $f(x) = \left(\frac{1}{3}\right)^{x-2} - 1$.

   (a) Applying translations to the graph of the function $\left(\frac{1}{3}\right)^x$, Sketch the graph of $f(x)$.

   (b) Find the x- and y-intercepts of $f(x)$.

3. Given $f(x) = -\left(\frac{2}{3}\right)^x + 2$.

   (a) Decide whether f is an increasing or a decreasing function.

   (b) Find the y-intercept of f.

4. Simplify the expression $(e^x + e^{-x})^4 - (e^x - e^{-x})^4$.

5. Find the equation of the form $y = a^x$ whose graph contains the point $(3, 8)$.

6. If $f(x) = a^x$ and $f(-2) = \frac{1}{3}$, then find $f(6)$.

7. If $f(t) = 3^{2-t}$ is written in the form $f(t) = ka^t$, then find the value of k and a.

8. Find the Domain and he Range of $f(x) = \frac{e^x + e^{-x}}{2}$.

9. Find the solution set of the inequality $x^2e^x - 2xe^x > 0$.

10. Given $f(x) = 3^{x+1}$ and $g(x) = \left(\frac{1}{3}\right)^{x+5}$, then which of the following statements is TRUE?

    (a) $g(x)$ is an increasing function.

    (b) $f(x)$ is an decreasing function.

    (c) The range of $f(x)$ is $[0, \infty)$.

    (d) The domain of $g(x)$ is $[0, \infty)$.

    (e) The graph of $f(x)$ and $g(x)$ intersect at $\left(-3, \frac{1}{9}\right)$.

2 Section 4.3

1. Let $f(x) = \log_{\frac{1}{2}}(3 - x)$.

   (a) Applying translations and reflections to the graph of the function $\log_{\frac{1}{2}} x$, sketch the graph of $f(x)$.

   (b) Find the Domain, the Range, and the Asymptote(s), if any, of the function $f(x)$.

   (c) Find the inverse function $f^{-1}(x)$.

2. For $a > 0$, $a \neq 1$, and $x > 1$, find the exponential form of the expression $y = \log_a(x - 1)$.

3. Given $f(x) = -\log_{\frac{1}{3}}(x+9) - 1$.

   (a) Find the x- and y-intercepts of $f(x)$.

   (b) Graph $f(x)$.

   (c) Find the domain and the range of $f(x)$.

   (d) Find the equation of the asymptote of the graph of $f(x)$.

4. Given $f(x) = -\frac{1}{2} + \log_9(1-2x)$.

   (a) Find the domain and the range of $f(x)$.

   (b) Find the asymptote (if any) of $f(x)$.

   (c) Find the x- and y-intercepts of $f(x)$.

5. Find the Domain and the Range of the function $y = -|\log_{\frac{1}{2}} x^2| + 1$.

6. The following figure is the graph of:

   (a) $f(x) = \log(x-1)$.

   (b) $f(x) = -\log|x|$.

   (c) $f(x) = \log(1-x)$.

   (d) $f(x) = \log(-x)$.

   (e) $f(x) = -\log(-x)$.

7. Find the domain of the function $y = \log\left(1 - x^2\right)$.

8. Find the x- and y-intercepts of $y = \log_3(2x+1) - 2$.

9. If $f(x) = \log(2x-1) - 3$, then find $f^{-1}(-2)$.

10. The following figure represents the graph of:

    (a) $y = \log_{\frac{1}{4}}(x-1)$.

    (b) $y = \log_{\frac{1}{4}}(x+1)$.

    (c) $y = 2^{-x+1} - 6$.

    (d) $y = 3^{-x+1} - 4$.

    (e) $y = -3^{-x+1} + 2$.

11. The following figure represents the graph of:

    (a) $y = x \ln x$.

    (b) $y = \frac{\ln x}{x}$.

    (c) $y = |\ln x|$.

    (d) $y = \ln|x|$.

(e) $y = \frac{x}{\ln x}$.

12. The following figure represents the graph of:

    (a) $y = \log_4 (x-2)$.
    (b) $y = \log_4 (2-x)$.
    (c) $y = \log_4 |2-x|$.
    (d) $y = \log_{\frac{1}{4}} (x-2)$.
    (e) $y = \left|\log_{\frac{1}{4}} (x-2)\right|$.

13. The following figure represents the graph of:

    (a) $y = \log_2 (x-1)$.
    (b) $y = \log_{\frac{1}{2}} (1-x)$.
    (c) $y = \log_2 (1-x)$.
    (d) $y = \log_{\frac{1}{2}} (x-1)$.
    (e) $y = \ln (1-x)$.

14. The following figure represents the graph of:

    (a) $y = \log_3 (2+x)$.
    (b) $y = \log_{\frac{1}{3}} (2-x)$.
    (c) $y = \log_3 (3-x)$.
    (d) $y = \log_{\frac{1}{3}} (3-x)$.
    (e) $y = \log_{\frac{1}{3}} (3+x)$.

15. The following figure represents the graph of:

    (a) $y = 1 + \log_2 |x-1|$.
    (b) $y = 1 + \log_2 |x-2|$.
    (c) $y = 1 + \log_2 \left|x - \frac{3}{2}\right|$.
    (d) $y = \log_2 |x-1|$.
    (e) $y = -1 + \log_2 |x-1|$.

16. Let $f(x)$ be a logarithmic function such that $f(2) = 3$. Find the value of $f(4)$.

3 Section 4.4

1. If $f(x) = e^x - e^{-x}$, then find the value of $f(2\ln 3)$.

2. Write the following as a single logarithmic function and simplify your answer if possible. (Assume $x > 0$, $y > 0$. and $z > 0$)

   (a) $3\log_2\left(y^2 z\right) - 2\log_2\left(xy^2\right) + \log_2\left(x^3 y z^4\right)$.

   (b) $1 + \log_2\left(x^2 y^3\right) - \frac{1}{2}\log_2\left(x^6 y^4\right)$.

   (c) $5\log_3 x - 8\log_9 y + \log_{\sqrt{3}} z + 1$. (with a base of 3).

   (d) $3\log_2 x - \log_{\sqrt{2}} y + \log_4 z^2$. (with a base of 2).

3. Find the value of the following:

   (a) $\ln\left(\frac{1}{e^3}\right) + e^{\ln 7}$.

   (b) $2\log_3\sqrt{18} - \log_3 2$.

   (c) $(\log_3 64)\left(\log_4\sqrt{3}\right) - \left(\sqrt[3]{10}\right)^{-3\log 5}$.

   (d) $\log_8 \frac{\sqrt[3]{16}}{4}$.

   (e) $[\log_9 35 - \log_9 7]\cdot[\log_5 9]$.

   (f) $\left[\sqrt{2}\right]^{\frac{\log 9}{\log 2}}$.

   (g) $(\log_5 16)\left(\log_2\sqrt{5}\right) - \left(\sqrt{e}\right)^{-6\ln 2}$.

   (h) $2\log 5 + \frac{1}{2}\log 16$.

   (i) $\ln(\ln e) + e^{-2\ln\sqrt{5}}$.

4. If $\log 2 = x$ and $\log 3 = y$, then write 1) $\log\left(\frac{9}{25}\right)$ 2) $\log 75$ in terms of x and y.

5. If $\log_2 5 = x$ and $\log_2 3 = y$, then write $\log_{\sqrt{2}} 300$ in terms of x and y.

6. If $\log_c 2 = \frac{2}{3}$, then find $\log_8 c$.

7. If $\ln 2 = x$ and $\ln 10 = y$, then write $\ln 16000 + \ln 5$ in terms of x and y.

8. If $\log 0.04 = x$, then write $\log 80$ in terms of x.

9. If $\ln 2 = 0.7$ and $\ln 3 = 1.1$, then find the value of 1) $\log_{36}\left(\frac{e^3}{12}\right)$ 2) $\log_{\frac{2}{3}} \frac{4e^2}{27}$.

10. If $\log x = 2$, $\log y = 3$, and $\log z = 5$, then find $\log \frac{x^3 y}{\sqrt{z}} - \log_x z$.

11. If $\log 2 = a$, and $\log 3 = b$, then write $\log_4 60$ in terms of a and b.

12. If $\log_3 a = \frac{1}{3}$, then find $\log_a\left(\frac{1}{9}\right)$.

13. If $\ln 2 = x$ and $\ln 6 = y$, then write $\log_9 4$ in terms of x and y.

14. If $\log_2(x-1) = \frac{1}{2}$, then find the value of $\log_2\left(2x^2 - 4x + 2\right)$.

15. If $a > 0$, $a \neq 1$, and $y = \frac{\log(\ln a)}{\log a}$, then find a^y.

16. Find the value of $\ln \ln e^{e^{x+3}} - e^{\ln x}$.

17. Write $\log_8 e^3 x$ in terms of $\ln x$.

18. Write $\log_a \frac{1}{x}$ in terms of log with base $\frac{1}{a}$.

19. Which one of the following is FALSE?

(a) $\ln e^x = x$ for any real number x.

(b) $e^{\ln x} = x$ for any real number x.

(c) $\ln \frac{1}{10} < \ln \frac{1}{3}$.

(d) $\log_{\frac{1}{3}} 4 > \log_{\frac{1}{3}} 5$.

(e) $g(x) = \left(\frac{1}{3}\right)^{-x}$ is an increasing function.

20. If $x > 0$, then which one of the following is TRUE?

(a) $\log(1 + x) = \frac{x}{1+x}$.

(b) $\log(1 + x) < \frac{x}{1+x}$.

(c) $\log(1 + x) > x$.

(d) $\log(1 + x) < x$.

(e) none of the above.

21. Which one of the following is FALSE?

(a) $\log_{\frac{1}{2}} 8 = -3$.

(b) $\log_a xy = \log_a x + \log_a y$, $x > 0$, $y > 0$, $a > 0$, and $a \neq 1$.

(c) $y = \log_a x$ if and only if $x = a^y$, $x > 0$, $a > 0$, and $a \neq 1$.

(d) $a^{\log_a x} = x$, $x > 0$, $a > 0$, and $a \neq 1$.

(e) $\frac{\log_a x}{\log_a y} = \log_a (x - y)$.

22. Find the solution set of the following inequalities:

(a) $\log(x + 4) < 0$.

(b) $\log_3 x + 2\log_9 x > 2$.

(c) $\log_{\frac{1}{2}} x^2 > -4$.

(d) $\log_5 x < \log x$.

(e) $\log_x 64 < 3$.

(f) $\log_2 x < -1$.

4 Section 4.5

1. Find the solution set of the following equations:

   (a) $(125)^{3-x} = (25)^x 5^{1-x}$.

   (b) $8^{2x-1} = 2\left(\frac{1}{16}\right)^{-\frac{1}{2}}$.

   (c) $4^x - 7 \cdot 2^x + 12 = 0$.

   (d) $\left(\frac{2}{3}\right)^{|k-5|} = \left(\frac{81}{16}\right)^{-|k|}$.

   (e) $\frac{5^x+5^{-x}}{5^x-5^{-x}} = 3$.

   (f) $\frac{10^x-(200)\left(10^{-x}\right)}{2} = 49$.

   (g) $4^x - (3)(2^x) + 2 = 0$.

   (h) $(343)^{3-x} = (49)^x$.

   (i) $\left(\frac{3}{2}\right)^{|2x-1|} = \frac{27}{8}$.

   (j) $\left(\frac{2}{3}\right)^{x-2} = \left(\frac{27}{8}\right)^{-2(x+3)}$.

   (k) $e^x - 5 + 6e^{-x} = 0$.

   (l) $2^{2x} + 6 \cdot 2^x + 4 = 0$.

   (m) $2^{2x+1} - 7 \cdot 2^x - 4 = 0$.

   (n) $(125)^{x(x-5)} = \left(\frac{1}{25}\right)^3$.

   (o) $(e^x - 3)(e^x + 1) = -3$.

2. Find the solution set of the following equations:

   (a) $\frac{1}{3}\log_2(x+5) + \log_8(3x-1) = 2$.

   (b) $\log(x-2) + \log(x+1) = 1$.

   (c) $2\log(x-2) = \log(\log 10^x) + 10^{\log(\log x)}$.

   (d) $\log_8(x+5) + \log_8(3x-1) = \log_4 16$.

   (e) $2\log_3(1-x) + \log_{\frac{1}{3}}(x-2) = 2\log_3 2$.

   (f) $\log_3(-x) + \log_3(6-x) = 3$.

   (g) $5\log_2(\log_4 16) + x = 1 + 2\ln e^x$.

   (h) $\log_5(x-20) - \log_5 \frac{1}{x} = \log 1000$.

   (i) $\log x^3 = (\log x)^2$.

   (j) $\log_3\left(\log_{\frac{1}{2}} x\right) = 1$.

   (k) $(\log_c x) \cdot \log_5 c = 3$.

   (l) $\log_x(\log_2 8) = 2$.

   (m) $\log_{\frac{1}{x}} x^2 = 6$.

(n) $2\log_2 x - \log_2(x-1) = 2$.

(o) $2\log(x-3) = \log(x+5) + \log 4$.

(p) $(\log x)^2 + 2 - \log x^3 = 0$.

(q) $\log(3x-1) = 1 - \log x$.

(r) $2\log\left(\sqrt{x+3}\right) + \log(2-x) = \log(-2x)$.

(s) $\ln x = -(\ln x)^2$.

(t) $\log_{3x+1} 4 = -2$.

(u) $\ln(x-2) - \log_{e^{-1}}(x+2) - \ln\left(e^{\ln 12}\right) = 0$.

(v) $\log_2 \sqrt{x-2} + \log_4(x-4) = \frac{1}{2}(3 + \log_2 3)$.

(w) $(\ln x)^2 - \ln x^3 + 2 = 0$.

3. Find the product of all solutions of the equation $\log(2-6x) + \log(8+x) = 2$.

4. Find the value of y if $y^{\frac{1}{3}} = \log_{\frac{1}{10}} 100$.

5. let $\ln 2 = x$ and $\ln 3 = y$. If $2^{t+1} = 3^{2t-1}$, then write t in terms of x and y.

6. If $y = \ln(x-3) + 1$, then write x in terms of y.

7. If $t = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$, then write x in terms of t.

8. In the formula $P(t) = P_0 e^{kt}$, if $P(25) = \frac{1}{2}P_0$, then find $P(75)$ in terms of P_0.

9. If $\log_2 6x - \log_2 3x = 2\log_2 k$, and $x > 0$, then find k.

10. Find the points of intersection of the graphs of $f(x) = e^{x^2}$ and $g(x) = (e^x)^2$.