Math 002 – Term 072
Recitation Hour (5.3 & 5.4)

Q1) if the terminal side of an angle θ passes through the point $(-5,-12)$,

then find the value of $\dfrac{\sec\theta-\tan\theta}{\cos\theta+\sin\theta}$.

Solution:

Notice that θ lies in quadrant III. Now $x=-5, y=-12 \implies r=\sqrt{x^2+y^2}=\sqrt{25+144}=\sqrt{169}$ $=13 \implies \sec\theta=\dfrac{r}{x}=-\dfrac{13}{5}$, $\tan\theta=\dfrac{y}{x}=\dfrac{12}{5}$, $\cos\theta=\dfrac{x}{r}=-\dfrac{5}{13}$ and $\sin\theta=\dfrac{y}{r}=-\dfrac{12}{13}$.

The value $=\dfrac{-\frac{13}{5}-\frac{12}{5}}{-\frac{5}{13}-\frac{12}{13}}=\dfrac{\frac{-25}{5}}{\frac{-17}{13}}=\dfrac{(5)(13)}{17}=\frac{65}{17}$

Q2) (a) Find the reference angle θ' for the following angle θ: (i) -570° (ii) 30 radians

(b) Find the exact value of:

(i) $\cos 44^\circ+\cos 136^\circ+\sin(-510^\circ)$ (ii) $\cot\dfrac{31\pi}{4}-\cos\dfrac{23\pi}{3}-\csc 570^\circ$

Solution:

(a) (i) The smallest positive angle coterminal with -570° is $-570^\circ+2(360^\circ)=-570^\circ+720^\circ$

$=150^\circ$ which lies in quadrant II $\implies \theta'=180^\circ-150^\circ=30^\circ$

(ii) Clearly $10\pi<\theta=30$ radians $<9\pi \implies$ the smallest positive angle coterminal with θ is $30-8\pi$ ($\approx 30-25.12=4.88$) which lies in quadrant IV $\implies \theta'=2\pi-(30-8\pi)$ $=10\pi-30$ ($\approx 31.4-30=1.4$)

(b) (i) $\theta=136^\circ$ lies in quadrant II $\implies \theta'=180^\circ-136^\circ=44^\circ$. The smallest positive angle coterminal with $\theta=-510^\circ$ is $-510^\circ+720^\circ=210^\circ$ which lies in quadrant III $\implies \theta'=210^\circ-180^\circ=30^\circ$. Therefore the value $=\cos 44^\circ+(-\cos 44^\circ)+(-\sin 30^\circ)=\cos 44^\circ-\cos 44^\circ-\dfrac{1}{2}=-\dfrac{1}{2}$

(ii) Using the coterminal and refrence angles again: $\cot\dfrac{31\pi}{4}-\cos\dfrac{23\pi}{3}-\csc 570^\circ$

$=\cot(\dfrac{31\pi}{4}-6\pi)-\cos(\dfrac{23\pi}{3}-6\pi)-\csc(570^\circ-360^\circ)=\cot\dfrac{7\pi}{4}-\cos\dfrac{5\pi}{3}-\csc 210^\circ$

$=-(\cot\dfrac{\pi}{4})-(\cos\dfrac{\pi}{3})-(-\csc 30^\circ)=-1-\dfrac{1}{2}+2=\dfrac{1}{2}$

Continue

Q3) (a) If W is the wrapping function, then find $W\left(-\frac{19\pi}{6}\right)$.

(b) If $\cos 170^\circ = k$, then find the value $\cos 350^\circ + 2\sec 190^\circ$ in terms of k.

Solution:

(a) $W(-\frac{19\pi}{6}) = (x,y) = \left(\cos(-\frac{19\pi}{6}), \sin(-\frac{19\pi}{6})\right) = \left(\cos(-\frac{19\pi}{6}+4\pi), \sin(-\frac{19\pi}{6}+4\pi)\right)$

$= \left(\cos\frac{5\pi}{6}, \sin\frac{5\pi}{6}\right) = \left(-\cos\frac{\pi}{6}, \sin\frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(b)

θ	Quadrant	θ'
170°	II	$180^\circ - 170^\circ = 10^\circ$
350°	IV	$360^\circ - 350^\circ = 10^\circ$
190°	III	$190^\circ - 180^\circ = 10^\circ$

Now $\cos 170^\circ = k \implies -\cos 10^\circ = k \implies \cos 10^\circ = -k$ and $\sec 10^\circ = -\frac{1}{k}$.

Therefore $\cos 350^\circ + 2\sec 190^\circ = \cos 10^\circ + 2(-\sec 10^\circ) = -k - 2\left(-\frac{1}{k}\right) = -k + \frac{2}{k} = \frac{2-k^2}{k}$

Q4) (a) Write $\csc t$ in terms of $\tan t$, where $\pi < t < \frac{3\pi}{2}$

(b) Determine whether the function $f(x) = x\sin x - \cos x$ is even, odd, or neither.

Solution:

(a) Since t lies in quadrant III, then $\boxed{\csc t < 0}$. Now $\csc^2 t = 1 + \cot^2 t = 1 + \frac{1}{\tan^2 t}$

$= \frac{\tan^2 t + 1}{\tan^2 t} \implies \csc t = -\sqrt{\frac{\tan^2 t + 1}{\tan^2 t}} = -\frac{\sqrt{1+\tan^2 t}}{|\tan t|} = -\frac{\sqrt{1+\tan^2 t}}{\tan t}$

(b) $f(-x) = (-x)[\sin(-x)] - \cos(-x) = (-x)(-\sin x) - \cos x = x\sin x - \cos x = f(x)$.

This means that $f(x)$ is an even function.

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