Math 002 – Term 072
Recitation Hour (4.4 & 4.5)

Q1) Find the value of: (a) $(\log_9 35 - \log_9 7)(\log_5 9)$ (b) $\ln\left(\frac{1}{e^3}\right) + e^{-\ln 7}$

Solution: (a) $= \left(\log_9 \frac{35}{7}\right)(\log_5 9) = (\log_9 5)(\log_5 9) = \frac{\log_5 5}{\log_5 9} \cdot \log_5 9 = 1$

(b) $= \ln e^{-3} + e^{\ln 7^{-1}} = -3 + \frac{1}{7} = -\frac{20}{7}$

Q2) Write the expression as a single logarithm with a base of 3: $5\log_3 x - 8\log_9 y + \log_{\sqrt{3}} z + 1$

Solution: $= \log_3 x^5 - \frac{\log_3 y^8}{\log_3 9} + \frac{\log_3 z}{\log_3 \sqrt{3}} + \log_3 3 = \log_3 x^5 - \frac{1}{2}\log_3 y^8 + 2\log_3 z + \log_3 3 =$

$\log_3 x^5 - \log_3 y^4 + \log_3 z^2 + \log_3 3 = \log_3(3\, x^5\, z^2) - \log_3 y^4 = \log_3\left(\frac{3\, x^5\, z^2}{y^4}\right)$

Q3) If $\log x = a$, $\log y = b$, then express $\log_x(x^2\sqrt{y})$ in terms of a and b.

Solution: $= \log_x x^2 + \log_x \sqrt{y} = 2\log_x x + \frac{\log\sqrt{y}}{\log x} = 2(1) + \frac{\frac{1}{2}\log y}{\log x} = 2 + \frac{b}{2a} = \frac{4a+b}{2a}$

Q4) Solve the following equations:

(a) $\log_5(x-20) - \log_5 \frac{1}{x} = \log 1000$ (b) $\frac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

Solution: (a) $\log_5(x-20) - \log_5 x^{-1} = \log 10^3 = 3 \implies \log_5(x-20) + \log_5 x = 3 \implies$

$\log_5 x(x-20) = 3 \implies x^2 - 20x = 5^3 = 125 \implies x^2 - 20x - 125 = 0 \implies$

$(x-25)(x+5) = 0 \implies x = 25$ or $x = -5$. Next we must check our answer:

For $x = 25$: $\log_5(5) - \log_5(\frac{1}{25}) \stackrel{?}{=} \log 1000 = 3 \implies 1 - (-2) \stackrel{?}{=} 3$ (yes)

For $x = -5$: $\log_5(-5-20)$ is undefined, so $x = -5$ is rejected. Thus S.S. $= \{25\}$

(b) $\frac{e^x + e^{-x}}{e^x - e^{-x}} \cdot \frac{e^x}{e^x} = 3 \implies \frac{e^{2x}+1}{e^{2x}-1} = 3 \implies 3e^{2x} - 3 = e^{2x} + 1 \implies 2e^{2x} = 4$

$\implies e^{2x} = 2 \implies 2x = \ln 2 \implies x = \frac{1}{2}\ln 2 = \ln\sqrt{2}$

Done By: A. Al-Shallali