Math 002 – Term 072
Recitation hour (4.2 & 4.3)

Q1) For the function $f(x) = -\,2^{2-x} + 4$,

(a) Find $x-$ and $y-$intercepts (b) Sketch the graph of $f(x)$ (c) Find the range of $f(x)$

(d) Find the asymptote(s) (e) Find $f^{-1}(x)$

Solution: $f(x) = -\,2^{2-x} + 4 = -\left(\frac{1}{2}\right)^{x-2} + 4$

(a , b) First, we draw $\boxed{y = 2^{-x} = (\frac{1}{2})^x}$ (which is decreasing, because $0 <$ base $= \frac{1}{2} < 1$)

Next, shift this graph 2 units to the right to get $\boxed{y = 2^{2-x}}$ with $y-$ intercept $= (0, 4)$. Then reflect the last graph across $x-$ axis to get $\boxed{y = -\,2^{2-x}}$ with $y-$ intercept $= (0, -4)$. Finally shift the latest one 4 units upward to get $\boxed{f(x) = -\,2^{2-x} + 4}$

From the graph of $f(x)$: $x-$ intercept $= y-$ intercept $= (0, 0)$

(c) The range $= (-\infty, 4)$,

(d) The horizental asymptote is $y = 4$.

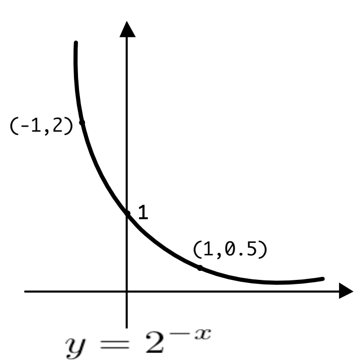
(e) To find $f^{-1}(x)$:

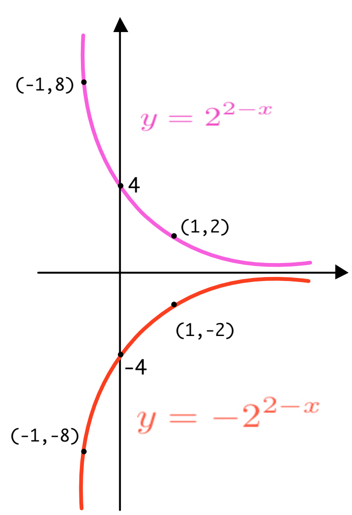
interchange x with y, so $x = -2^{2-y} + 4 \implies$

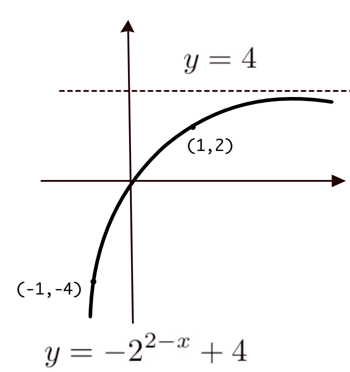
$2^{2-y} = 4 - x \implies 2 - y = \log_2(4 - x) \implies$

$y = f^{-1}(x) = 2 - \log_2(4 - x)$.

Notice that $D_{f^{-1}} = (-\infty, 4) = R_f$







Q2) For the function $f(x) = -\ \log_2(x+4)$,

(a) sketch the graph of $f(x)$ (b) Find the $x-$ and $y-$ intercepts (c) The domain

(d) Find the asymptote(s) (e) Sketch $g(x) = -\log_2|x+4|$.

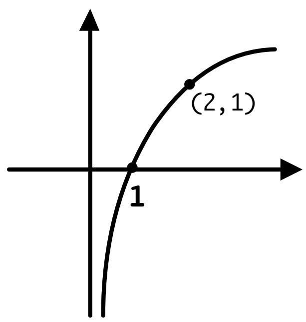
<u>**Solution**</u>: (a) First, we draw $\boxed{y = \log_2 x}$ (which is increasing because the base $= 2 > 1$) and has an x–intercept = (1,0). Next, shift the first graph 4 units to the left to get $\boxed{y = \log_2(x+4)}$, then reflect the second graph across the x–axis to get $\boxed{f(x) = -\log_2(x+4)}$.

b) From the graph of $f(x)$: $x-$ intercept $= (-3, 0)$, $y-$ intercept $= (0, -2)$.

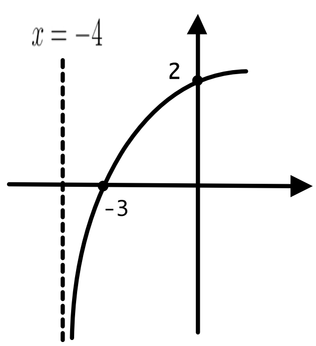
c) The domain $= (-4, \infty)$ d) The vertical asymptote is $x = -4$.

e) Add to the graph of $y = -\log_2(x+4)$ its reflection across its asymptote $x = -4$ to get

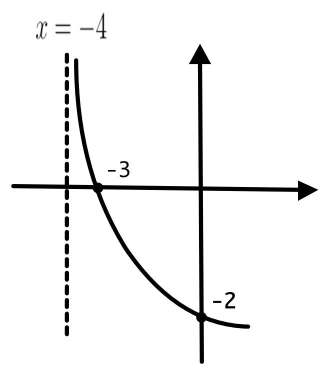
$\boxed{g(x) = -\log_2|x+4|}$



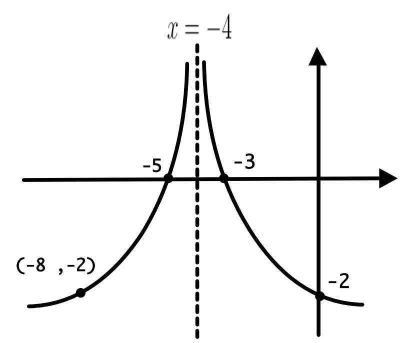
$y = \log_2 x$



$y = \log_2(x+4)$



$f(x) = -\log_2(x+4)$



$g(x) = -\log_2|x+4|$

Q3) If the graph of the logarithmic function $f(x) = \log_b x$ passes through the point $(\frac{1}{64}, -3)$, then find $f(2)$.

<u>**Solution**</u>: $-3 = \log_b \frac{1}{64} = \log_b \left(\frac{1}{2}\right)^6 = \log_b 2^{-6} = -6 \log_b 2 \implies \log_b 2 = \frac{-3}{-6} = \frac{1}{2}$

$\implies f(2) = \log_b 2 = \frac{1}{2}$

Q4) If a bacteria population starts with 1000 bacteria and doubles every three hours, then the number of bacteria after t hours is $N(t) = 1000 \cdot 2^{t/3}$. When will the population reach 64000?

<u>**Solution**</u>: $64000 = 1000 \cdot 2^{t/3} \implies 64 = 2^{t/3} \implies 2^6 = 2^{t/3} \implies 6 = \frac{t}{3} \implies t = 18$ hours.

Done By: A. Al-Shallali