

## Math 002 – Term 052 Recitation hour (4.2 & 4.3)

**Q1)** For the function  $f(x) = -\left(\frac{1}{2}\right)^{-x+2} - 2$ , sketch the graph of  $f(x)$ , find the  $x$ - and  $y$ - intercepts, the range, asymptote(s), and then find  $f^{-1}(x)$ .

**Solution:**  $f(x) = -\left(\frac{1}{2}\right)^{-x+2} - 2 = -2^{x-2} - 2$

First, we draw  $y = 2^x$  (which is increasing because the base = 2 > 1)

Next, shift this graph 2 units to the right to get  $y = 2^{x-2}$  with  $y$ - intercept =  $(0, \frac{1}{4})$ . Then reflect the last graph across  $x$ - axis to get

$y = -2^{x-2}$  with  $y$ - intercept =  $(0, -\frac{1}{4})$ . Finally shift the latest one

2 units downward to get  $f(x) = -2^{x-2} - 2$

From the graph of  $f(x)$  :  $y$ - intercept =  $(0, -\frac{9}{4})$ ,

the range =  $(-\infty, -2)$ , and the horizontal

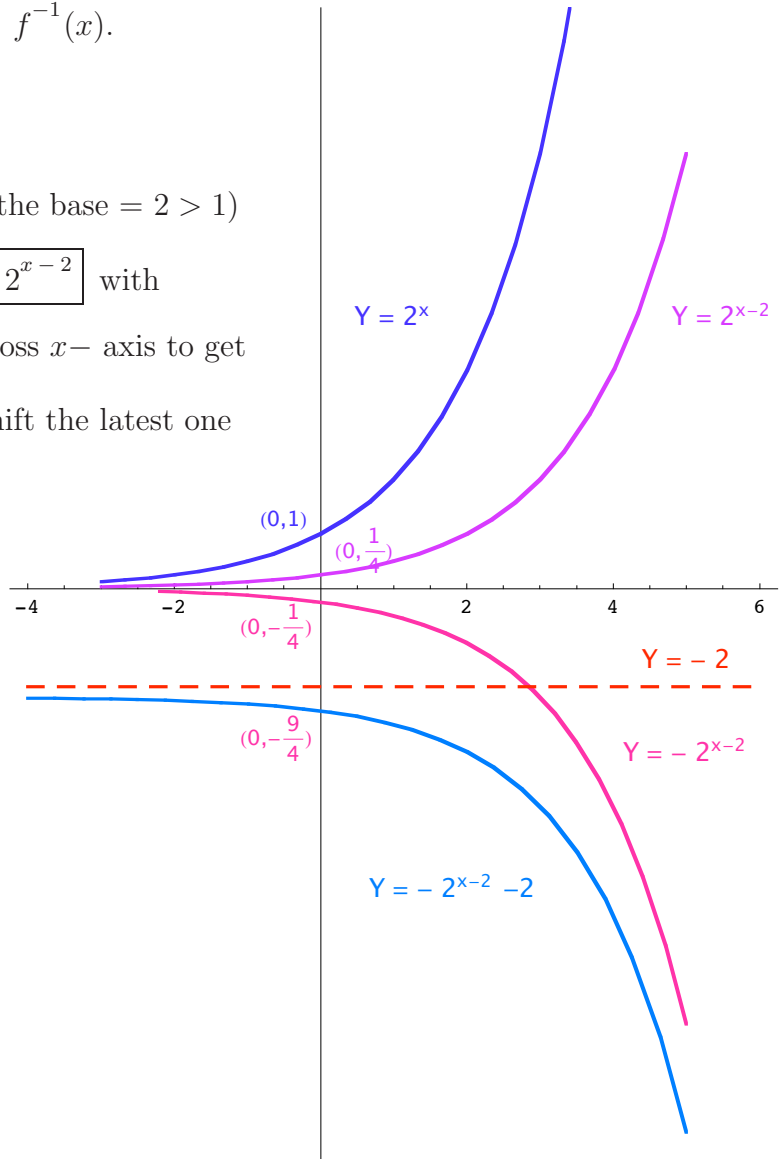
asymptote is  $y = -2$ . To find  $f^{-1}(x)$ :

interchange  $x$  with  $y$ , so  $x = -2^{y-2} - 2 \implies$

$2^{y-2} = -x - 2 \implies y - 2 = \log_2(-x - 2) \implies$

$y = f^{-1}(x) = 2 + \log_2(-x - 2)$ .

Notice that  $D_{f^{-1}} = (-\infty, -2) = R_f$



**Q2)** For the function  $f(x) = -\log_2(3-x)$ , sketch the graph of  $f(x)$ , find the  $x$ - and  $y$ - intercepts, the domain, asymptote(s), and then find  $f^{-1}(x)$ .

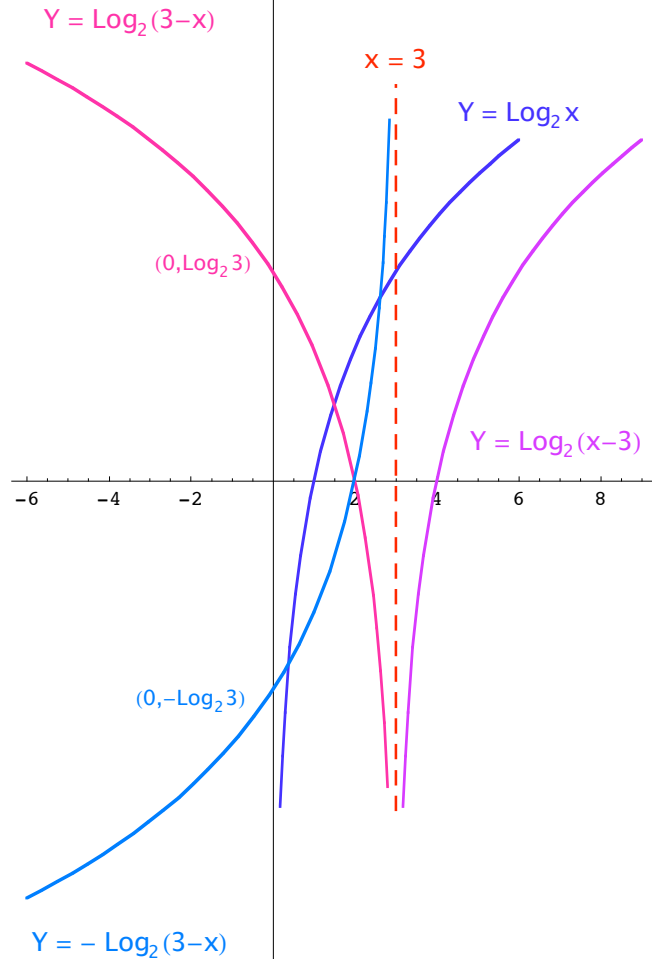
**Solution:** (a) First, we draw  $y = \log_2 x$  (which is increasing because the base = 2 > 1) and has an  $x$ -intercept =  $(1, 0)$

(b) Next, shift graph (a) 3 units to the right to get  $y = \log_2(x-3)$  with  $x$ - intercept =  $(4, 0)$ , vertical asymptote  $x = 3$ .

(c) Then reflect graph (b) across the line  $x = 3$  to get  $y = \log_2(3-x)$  with  $x$ - intercept =  $(2, 0)$ , and  $y$ - intercept =  $(0, \log_2 3)$ .

(d) Finally reflect graph (c) across  $x$ - axis to get  $f(x) = -\log_2(3-x)$

From the graph of  $f(x)$  :  $x$ - intercept =  $(2, 0)$ ,  
 $y$ - intercept =  $(0, -\lg_2 3) = (0, \log_2 \frac{1}{3})$ ,  
the domain =  $(-\infty, 3)$ , and the vertical asymptote  
is  $x = 3$ . To find  $f^{-1}(x)$ : interchange  $x$  with  $y$ ,  
so  $x = -\log_2(3 - y) \implies -x = \log_2(3 - y) \implies$   
 $3 - y = 2^{-x} \implies y = f^{-1}(x) = 3 - 2^{-x}$ .  
Notice that  $R_{f^{-1}} = (-\infty, 3) = D_f$



**Q3)** If the graph of the logarithmic function  $f(x) = \log_b x$  passes through the point  $(\frac{1}{64}, -3)$ , then find  $f(2)$ .

**Solution:**  $-3 = \log_b \frac{1}{64} = \log_b \left(\frac{1}{2}\right)^6 = \log_b 2^{-6} = -6 \log_b 2 \implies \log_b 2 = \frac{-3}{-6} = \frac{1}{2}$   
 $\implies f(2) = \log_b 2 = \frac{1}{2}$

**Q4)** If a bacteria population starts with 1000 bacteria and doubles every three hours, then the number of bacteria after  $t$  hours is  $N(t) = 1000 \cdot 2^{t/3}$ . When will the population reach 128000?

**Solution:**  $128000 = 1000 \cdot 2^{t/3} \implies 128 = 2^{t/3} \implies 2^7 = 2^{t/3} \implies 7 = \frac{t}{3} \implies t = 21$  hours.