Math 002 – Term 052
Recitation hour (4.4 & 4.5)

Q1) Expand the logarithm $\log \sqrt[5]{\dfrac{x^2 z^2}{y^4+2}}$, where, $x>0, z>0$

Solution: $= \frac{1}{5}\Big[\log(x^2 z^2) - \log(y^4+2)\Big] = \frac{1}{5}\Big[\log x^2 + \log z^2 - \log(y^4+2)\Big]$

$= \frac{1}{5}\Big[2\log x + 2\log z - \log(y^4+2)\Big] = \frac{2}{5}\log x + \frac{2}{5}\log z - \frac{1}{5}\log(y^4+2)$

Q2) If $\log 2 = 0.3$ and $\log 3 = 0.5$, then find the value of (i) $\log 6$ (ii) $\log 5$ (iii) $\log_5 600$

Solution: (i) $\log 6 = \log(2)(3) = \log 2 + \log 3 = 0.3 + 0.5 = 0.8$

(ii) $\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0.3 = 0.7$

(iii) $\log_5 600 = \dfrac{\log 600}{\log 5} = \dfrac{\log 100 + \log 6}{\log 5} = \dfrac{2+0.8}{0.7} = \dfrac{2.8}{0.7} = \dfrac{28}{7} = 4$

Q3) Find the exact values of (i) $\dfrac{\log_5 32}{\log_5 2}$ (ii) $\ln(\ln e) + e^{(1-2\ln 3)}$ (iii) $(\sqrt{3})^{\frac{1}{\log_4 3}}$

Solution: (i) $\dfrac{\log_5 32}{\log_5 2} = \log_2 32 = \log_2 2^5 = 5\log_2 2 = 5$

(ii) $\ln(\ln e) + e^{(1-2\ln 3)} = \ln 1 + e \cdot e^{-2\ln 3} = 0 + (e)(3)^{-2} = \dfrac{e}{9}$

(iii) $(\sqrt{3})^{\frac{1}{\log_4 3}} = \left(3^{\frac{1}{2}}\right)^{\log_3 4} = 3^{\frac{1}{2}\log_3 4} = 3^{\log_3 \sqrt{4}} = \sqrt{4} = 2$ (notice that $\dfrac{1}{\log_a b} = \log_b a$)

Q4) Solve the following equations: (i) $\left(\dfrac{1}{9}\right)^{(3-x)} = (81)^{(2x-5)}$ (ii) $\log_3(-x) + \log_3(6-x) = 3$

Solution: (i) $\left(\dfrac{1}{9}\right)^{(3-x)} = (81)^{(2x-5)} \implies 3^{-2(3-x)} = 3^{4(2x-5)} \implies -6+2x = 8x-20$

$\implies 14 = 6x \implies x = \dfrac{14}{6} = \dfrac{7}{3} \implies$ S.S. $= \{\frac{7}{3}\}$

(ii) $\log_3(-x) + \log_3(6-x) = 3 \implies \log_3(-x)(6-x) = 3 \implies (-x)(6-x) = 3^3 = 27 \implies x^2 - 6x - 27 = 0 \implies (x-9)(x+3) = 0 \implies x = 9$ or $x = -3$

Checking for $x = 9$: $\log(-9)$ is undefined, so $x = 9$ is rejected.

for $x = -3$: $\log_3 3 + \log_3 9 \stackrel{?}{=} 3 \implies \log_3 27 \stackrel{?}{=} 3$ (yes). Therefore S.S. $= \{-3\}$

–Continue–

Q5) True or False :

(i) $\log x^2 = (\log x)^2$ is false, because $\log x^2 = \log[(x)(x)]$ while $(\log x)^2 = (\log x)(\log x)$ which are different.

(ii) $\log(3+3) = \log 3 + \log 3$ is is false, because $\log(3+3) = \log 6 \neq \log 3 + \log 3 = \log(3)(3) = \log 9$.

(iii) $\log(1+2+3) = \log 1 + \log 2 + \log 3$ is true, because $\log(1+2+3) = \log 6$ and
$\log 1 + \log 2 + \log 3 = 0 + \log 2 + \log 3 = \log(2)(3) = \log 6$

(iv) If x is any nonzero real number, then $\log x^2 = 2\log x$ is false, because when $x < 0$, $\log x^2$ is defined while $2\log x$ is not. (in fact, if $x \neq 0$, then $\log x^2 = 2\log|x|$)

(v) If $\log_{\frac{1}{2}} x > 0$, then $x \in (0,1)$ is true (see the graph of $y = \log_{\frac{1}{2}} x$)

Solved by: A. Al-shallali