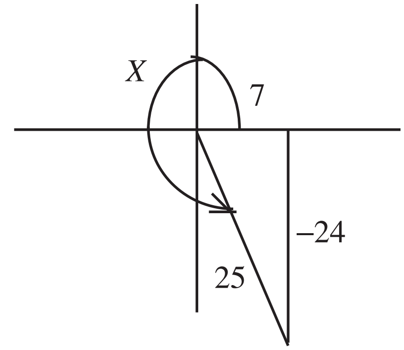
Math 002 – Term 052
Recitation hour (6.3–6.4)

Q1) If $\csc x = -\frac{25}{24}$, $\frac{3\pi}{2} < x < 2\pi$, then find $\tan 2x$, $\cos\frac{x}{2}$



Solution: $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x} = \dfrac{2(-\frac{24}{7})}{1-\left(-\frac{24}{7}\right)^2} = \dfrac{336}{527}$. Now

$\frac{3\pi}{2} < x < 2\pi \implies \frac{3\pi}{4} < \frac{x}{2} < \pi \implies \frac{x}{2} \in$ Q II $\implies$

$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+\frac{7}{25}}{2}} = -\sqrt{\frac{32}{50}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Q2) Find the exact value of: a) $\sin 112.5^\circ \cos 112.5^\circ$ b) $\sin\frac{\pi}{8}$

Solution: a) $= \frac{1}{2}(2\sin 112.5^\circ \cos 112.5^\circ) = \frac{1}{2}\sin 2(112.5^\circ) = \frac{1}{2}\sin 225^\circ = \frac{1}{2}(-\sin 45^\circ) = -\frac{1}{2}(\frac{\sqrt{2}}{2}) = -\frac{\sqrt{2}}{4}$

b) $\sin\frac{\pi}{8} = \sin\frac{1}{2}\left(\frac{\pi}{4}\right) = \sqrt{\dfrac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\dfrac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\dfrac{2-\sqrt{2}}{4}} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

Q3) Verify the identities: a) $\sin 2x - \tan x = \tan x \cos 2x$ b) $\sin^2\frac{x}{2}(1+\sec x)^2 = \cos^2\frac{x}{2}\tan^2 x$

Proof: a) L.H.S. $= 2\sin x\cos x - \tan x = \tan x\left(\dfrac{2\sin x\cos x}{\tan x} - 1\right) \tan x\left(\dfrac{2\sin x\cos x}{\frac{\sin x}{\cos x}} - 1\right)$

$= \tan x(2\cos^2 x - 1) = \tan x\cos 2x =$ L.H.S.

b) L.H.S. $= \left(\dfrac{1-\cos x}{2}\right)\left(1+\dfrac{1}{\cos x}\right)^2 = \left(\dfrac{1-\cos x}{2}\right)\left(\dfrac{1+\cos x}{\cos x}\right)^2$

$= \left(\dfrac{1+\cos x}{2}\right)(1-\cos x)\left(\dfrac{1+\cos x}{\cos^2 x}\right) = (\cos^2\frac{x}{2})\left(\dfrac{1-\cos^2 x}{\cos^2 x}\right)$

$= (\cos^2\frac{x}{2})\left(\dfrac{\sin^2 x}{\cos^2 x}\right) = \cos^2\frac{x}{2}\tan^2 x =$ R.H.S.

Q4) Given the function $f(x) = \sqrt{3}\sin 2x - \cos 2x$

a) Rewrite $f(x)$ in the form $f(x) = k\sin(2x+\alpha)$
b) Find the amplitude, the period, the phase shift, and the range of the graph of $f(x)$.

Solution: a) $\mathrm{A} = \sqrt{3}$, $\mathrm{B} = -1$, $b = 2$. So $k = \sqrt{\mathrm{A}^2+\mathrm{B}^2} = \sqrt{3+1} = 2$, $\cos\alpha = \dfrac{\mathrm{A}}{k} = \dfrac{\sqrt{3}}{2}$, $\sin\alpha = \dfrac{\mathrm{B}}{k}$ $= -\frac{1}{2} \implies \alpha \in$ Q IV, $\alpha' = \frac{\pi}{6} \implies \alpha = \frac{11\pi}{6} \implies f(x) = 2\sin(2x+\frac{11\pi}{6})$

b) Amplitude = k = 2, period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{2} = \pi$, phase shipt $= \dfrac{-\alpha}{b} = \dfrac{-\frac{11\pi}{6}}{2} = -\dfrac{11\pi}{12}$,

and range $= [-k, k] = [-2, 2]$

Notice that we can write $\alpha = -\frac{\pi}{6}$ instead of $\frac{11\pi}{6}$.