Math 002 – Term 052
Recitation hour (6.1–6.2)

Q1) Verify the following identities:

a) $\dfrac{\sin^2(-x)-\cos^2(-x)}{\sin(-x)-\cos(-x)} = \cos x - \sin x$ b) $\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \csc x - \cot x,\ 0 < x < \frac{\pi}{2}$

Solution: a) L.H.S. $= \dfrac{(-\sin x)^2 - \cos^2 x}{-\sin x - \cos x} = \dfrac{\sin x^2 - \cos^2 x}{-(\sin x + \cos x)} = \dfrac{(\sin x - \cos x)(\sin x + \cos x)}{-(\sin x + \cos x)}$

$= -(\sin x - \cos x) = \cos x - \sin x =$ R.H.S.

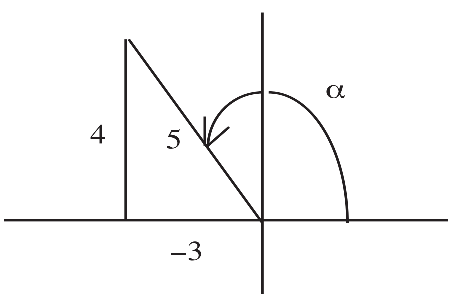
b) L.H.S. $= \sqrt{\dfrac{1-\cos x}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x}} = \sqrt{\dfrac{(1-\cos x)^2}{1-\cos^2 x}}\sqrt{\dfrac{(1-\cos x)^2}{\sin^2 x}} = \dfrac{|1-\cos x|}{|\sin x|} = \dfrac{1-\cos x}{\sin x}$

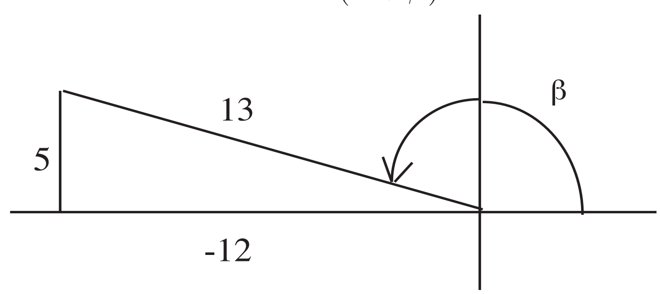
$= \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x} = \csc x - \cot x =$ R.H.S.

Q2) Given $\csc\alpha = \frac{5}{4}$, α in quadrant II, and $\cos(\frac{\pi}{2} - \beta) = \frac{5}{13}$, β in quadrant II, find $\csc(\alpha+\beta)$.

Solution: $\cos(\frac{\pi}{2}-\beta) = \frac{5}{13} \implies \sin\beta = \frac{5}{13}$. Now $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= (\frac{4}{5})(-\frac{12}{13}) + (-\frac{3}{5})(\frac{5}{13}) = -\frac{48}{65} - \frac{15}{65} = -\frac{63}{65} \implies \csc(\alpha+\beta) = \dfrac{1}{\sin(\alpha+\beta)} = -\frac{65}{63}$





Q3) Find the exact values of the following expressions:

a) $\cos 165^\circ$ b) $\sin 13^\circ \sin 73^\circ + \sin 77^\circ \sin 17^\circ$ c) $\dfrac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$

Solution: a) $\cos 165^\circ = \cos(45^\circ + 120^\circ) = \cos 45^\circ \cos 120^\circ - \sin 45^\circ \sin 120^\circ = \cos 45^\circ(-\cos 60^\circ) - \sin 45^\circ \sin 60^\circ$

$= (\frac{\sqrt{2}}{2})(-\frac{1}{2}) - (\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) = \dfrac{-\sqrt{2}-\sqrt{6}}{4} = -\dfrac{\sqrt{2}+\sqrt{6}}{4}$

b) $= \sin 13^\circ \cos(90^\circ - 73^\circ) + \cos(90^\circ - 77^\circ)\sin 17^\circ = \sin 13^\circ \cos 17^\circ + \cos 13^\circ \sin 17^\circ$

$= \sin(13^\circ + 17^\circ) = \sin 30^\circ = \frac{1}{2}$

c) $= \tan(69^\circ + 66^\circ) = \tan 135^\circ = -\tan 45^\circ = -1$

Q4) Prove that $\sin\alpha + \sin(120^\circ + \alpha) + \sin(240^\circ + \alpha) = 0$ for any angle α.

Proof: L.S. $= \sin\alpha + (\sin 120^\circ \cos\alpha + \cos 120^\circ \sin\alpha) + (\sin 240^\circ \cos\alpha + \cos 240^\circ \sin\alpha) = \sin\alpha + (\sin 60^\circ)\cos\alpha + (-\cos 60^\circ)\sin\alpha + (-\sin 60^\circ)\cos\alpha + (-\cos 60^\circ)\sin\alpha = \sin\alpha + \frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha - \frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha = \sin\alpha - \sin\alpha = 0 =$ R.S.