Math 002 – Term 052
Recitation hour (5.5–5.7)

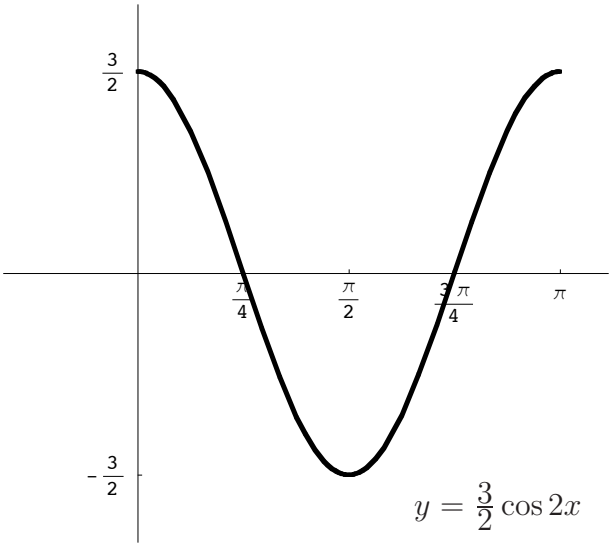
Q2) For the function $y = \frac{3}{2}\cos(2x - \frac{\pi}{2}) + 1$

a) Find the amplitude, period, phase shift, vertical translation, and the range.

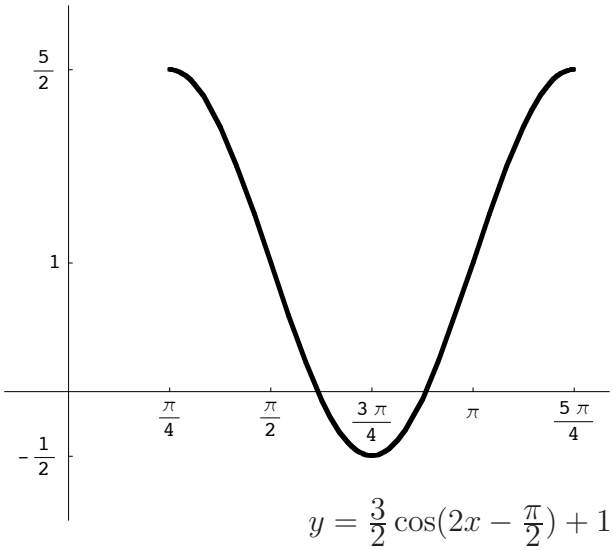
b) Graph the function over one period.

Solution: a) $a = \frac{3}{2}$, $b = 2$, $c = -\frac{\pi}{2}$, $d = 1$, so amplitude $= |a| = \frac{3}{2}$; period $= \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$. phase shift $= -\frac{c}{b} = -\frac{-\frac{\pi}{2}}{2} = \frac{\pi}{4}$ ($= \frac{\pi}{4}$ unit to the right) ; vertical translation $= |d| = 1$ unit upward ; range $= [-|a| + d, |a| + d] = [-\frac{3}{2} + 1, \frac{3}{2} + 1] = [-\frac{1}{2}, \frac{5}{2}]$.

b) First draw $y = \frac{3}{2}\cos 2x$ (which has amplitude $= \frac{3}{2}$ and period $= \pi$), then shift this graph $= \frac{\pi}{4}$ unit to the right and 1 unit upward to get $y = \frac{3}{2}\cos(2x - \frac{\pi}{2}) + 1$



$y = \frac{3}{2}\cos 2x$



$y = \frac{3}{2}\cos(2x - \frac{\pi}{2}) + 1$

Q3) For the function $f(x) = -2\tan(2x - \frac{\pi}{4})$, find

a) the period of $f(x)$.

b) the equation of all vertical asymptotes over the interval $[-\pi, \pi]$.

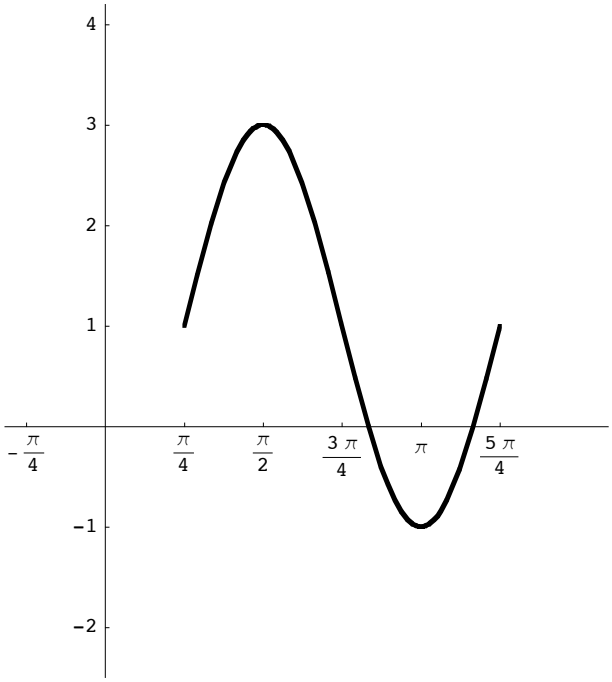
c) the x–intercepts over the interval $[-\pi, \pi]$.

Solution: a) The period $= \frac{\pi}{|b|} = \frac{\pi}{2}$

b) $f(x)$ has vertical asymptotes when $2x - \frac{\pi}{4} = (2n+1)\frac{\pi}{2} \implies 2x = n\pi + \frac{\pi}{2} + \frac{\pi}{4} = n\pi + \frac{3\pi}{4} = (4n+3)\frac{\pi}{4}$ $\implies x = (4n+3)\frac{\pi}{8}, n \in \mathrm{Z}$. For $n = -2$, $x = -\frac{5\pi}{8}$; $n = -1$, $x = -\frac{\pi}{8}$; $n = 0$, $x = \frac{3\pi}{8}$; $n = 1$, $x = \frac{7\pi}{8}$. Therefore the vertical asymptotesover the interval $[-\pi, \pi]$ are $x = -\frac{5\pi}{8}$, $x = -\frac{\pi}{8}$, $x = \frac{3\pi}{8}$, and $x = \frac{7\pi}{8}$

c) $f(x)$ has x–intercepts when $y = -2\tan(2x - \frac{\pi}{4}) = 0 \implies \frac{-2\sin(2x - \frac{\pi}{4})}{\cos(2x - \frac{\pi}{4})} = 0 \implies \sin(2x - \frac{\pi}{4}) = 0 \implies 2x - \frac{\pi}{4} = n\pi \implies 2x = n\pi + \frac{\pi}{4} = (4n+1)\frac{\pi}{4} \implies x = (4n+1)\frac{\pi}{8}, n \in \mathrm{Z}$. For $n = -2$, $x = -\frac{7\pi}{8}$; $n = -1$, $x = -\frac{3\pi}{8}$; $n = 0$, $x = \frac{\pi}{8}$; $n = 1$, $x = \frac{5\pi}{8}$.
Therefore all x–intercepts over the interval $[-\pi, \pi]$ are $(-\frac{7\pi}{8}, 0)$, $(-\frac{3\pi}{8}, 0)$, $(\frac{\pi}{8}, 0)$, and $(\frac{5\pi}{8}, 0)$.

Q4) The given graph represents the function $y = a\sin(bx + c) + d$. Find $a, b, c,$ and d.



Solution: Amplitude $= 3 - 1 = 2 \implies \boxed{a = 2}$. Period $= \frac{5\pi}{4} - \frac{\pi}{4} = \pi = \frac{2\pi}{b} \implies \boxed{\text{b=2}}$. Phase shift $= \frac{\pi}{4} = -\frac{c}{b} = -\frac{c}{2} \implies \boxed{c = -\frac{\pi}{2}}$. Range $= [-1, 3] = [-|a| + d, |a| + d] = [-2 + d, 2 + d] \implies \boxed{d = 1}$.

So $y = 2\sin(2x - \frac{\pi}{2}) + 1$