Math 002 — Fundamental Identities

- $\sin^2 x + \cos^2 x = 1 \Rightarrow \boxed{\sin^2 x = 1 - \cos^2 x}$, $\boxed{\cos^2 x - 1 = -\sin^2 x}$, $\boxed{\sin x = \pm\sqrt{1 - \cos^2 x}}$

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- Remember that in any **identity** you can **replace** x by any math. expression you like such as $3x$, $\frac{x}{2}$ or even $(2x^2 - 4)$. So we can get from the identity $\sin^2 x + \cos^2 x = 1$ as many identities as we like , e . g . , $\sin^2 4x + \cos^2 4x = 1$, $\sin^2 \frac{x}{5} + \cos^2 \frac{x}{5} = 1$ ... etc.
  But be careful and notice that : $2\sin^2 x + 2\cos^2 x = 2(\sin^2 x + \cos^2 x) = 2(1) = 2$
  and $-3\sin^2 4x - 3\cos^2 4x = -3(\sin^2 4x + \cos^2 4x) = -3(1) = -3$.

- $\sec^2 x = 1 + \tan^2 x \Rightarrow \boxed{\sec^2 x - \tan^2 x = 1}$, $\boxed{\tan^2 x - \sec^2 x = -1}$, $\boxed{\sec x = \pm\sqrt{1 + \tan^2 x}}$

- $\sec^2 x = 1 + \tan^2 x \Rightarrow \boxed{\tan^2 x = \sec^2 x - 1}$, $\boxed{1 - \sec^2 x = -\tan^2 x}$, $\boxed{\tan x = \pm\sqrt{\sec^2 x - 1}}$

- Again we can replace x in the identity $\sec^2 x = 1 + \tan^2 x$ by any math. expression to get identities like $\sec^2 3x = 1 + \tan^2 3x$, $\sec^2 \frac{x}{4} = 1 + \tan^2 \frac{x}{4}$ ... etc.
  While $2\sec^2 x - 2\tan^2 x = 2$, $3 - 3\sec^2 x = -3\tan^2 x$.

- $\csc^2 x = 1 + \cot^2 x \Rightarrow \boxed{\csc^2 x - \cot^2 x = 1}$, $\boxed{\cot^2 x - \csc^2 x = -1}$, $\boxed{\csc x = \pm\sqrt{1 + \cot^2 x}}$

- $\csc^2 x = 1 + \cot^2 x \Rightarrow \boxed{\cot^2 x = \csc^2 x - 1}$, $\boxed{1 - \csc^2 x = -\cot^2 x}$, $\boxed{\cot x = \pm\sqrt{\csc^2 x - 1}}$

- Similarly we have $\csc^2 5x - \cot^2 5x = 1$, $1 - \csc^2 \frac{x}{2} = -\cot^2 \frac{x}{2}$.
  While $6\cot^2 2x - 6\csc^2 2x = -6$, $4 - 4\csc^2 x = -4\cot^2 x$

- Notice that the $\pm$ in the above identities depends on the **position** of the angle x , e . g . ,
  $\sin x = \sqrt{1 - \cos^2 x}$ if x lies in the first or second quadrant
  $\sec x = -\sqrt{1 + \tan^2 x}$ if x lies in the second or third quadrant
  $\cot x = -\sqrt{\csc^2 x - 1}$ if x lies in the second or fourth quadrant

- Notice that each of the following is an identity :
  $\sin(-x)\ \csc(-x) = 1$
  $5\sin^2 4x + 5\cos^2 4x = 5$
  $\csc x = \sqrt{1 + \cot^2 x}\ ;\ 0 < x < \pi$
  $\sec\sqrt{x-3}\ \cos\sqrt{x-3} = 1$
  $\tan(x+2) = \pm\sqrt{\sec^2(x+2) - 1}$
  $\sec x = -\sqrt{1 + \tan^2 x}\ ;\ \frac{\pi}{2} < x < \frac{3\pi}{2}$

- While each of the following is **not** an identity :
  $\sin\ (\csc x) = 1$
  $\sin x + \cos x = 1$
  $\sec x = \sqrt{1 + \tan^2 x}$
  $\sin^2 10x + \cos^2 10x = 10$
  $\tan(-x)\ \cot x = 1$
  $\csc x = 1 + \cot x$
  $\cot x = -\sqrt{\csc^2 x - 1}$
  $\sec^2 3x - \tan^2 3x = 3$

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