# Math 002–Trigonometry

- Expressing a trigonometric function in terms of another trigonometric function:

To do so, we need to use the following identities:

1. $\tan x = \dfrac{\sin x}{\cos x}$ , $\cot x = \dfrac{\cos x}{\sin x}$
2. $\csc x = \dfrac{1}{\sin x}$ , $\sec x = \dfrac{1}{\cos x}$ , $\cot x = \dfrac{1}{\tan x}$
3. $\sin^2 x + \cos^2 x = 1$
4. $\sec^2 x = 1 + \tan^2 x$
5. $\csc^2 x = 1 + \cot^2 x$

Notice that $\sin x = \sqrt{1-\cos^2 x}$ is not always true, because $\sin x$ is sometimes negative while $\sqrt{1-\cos^2 x}$ is never negative. The correct identity is $\sin x = \pm\sqrt{1-\cos^2 x}$. Similarly $\sec x = \pm\sqrt{1+\tan^2 x}$ and $\cot x = \pm\sqrt{\csc^2 x - 1}$ are both identities. Here is the whole table:

| $f(x)$ | $\sin x$ | $\cos x$ | $\tan x$ | $\cot x$ | $\sec x$ | $\csc x$ |
|---|---|---|---|---|---|---|
| $\sin x =$ | $\sin x$ | $\pm\sqrt{1-\cos^2 x}$ | $\dfrac{\pm\tan x}{\sqrt{1+\tan^2 x}}$ | $\dfrac{\pm 1}{\sqrt{1+\cot^2 x}}$ | $\dfrac{\pm\sqrt{\sec^2 x - 1}}{\sec x}$ | $\dfrac{1}{\csc x}$ |
| $\cos x =$ | $\pm\sqrt{1-\sin^2 x}$ | $\cos x$ | $\dfrac{\pm 1}{\sqrt{1+\tan^2 x}}$ | $\dfrac{\pm\cot x}{\sqrt{1+\cot^2 x}}$ | $\dfrac{1}{\sec x}$ | $\dfrac{\sqrt{\csc^2 x - 1}}{\csc x}$ |
| $\tan x =$ | $\dfrac{\pm\sin x}{\sqrt{1-\sin^2 x}}$ | $\dfrac{\pm\sqrt{1-\cos^2 x}}{\cos x}$ | $\tan x$ | $\dfrac{1}{\cot x}$ | $\pm\sqrt{\sec^2 x - 1}$ | $\dfrac{\pm 1}{\sqrt{\csc^2 x - 1}}$ |
| $\cot x =$ | $\dfrac{\pm\sqrt{1-\sin^2 x}}{\sin x}$ | $\dfrac{\pm\cos x}{\sqrt{1-\cos^2 x}}$ | $\dfrac{1}{\tan x}$ | $\cot x$ | $\dfrac{\pm 1}{\sqrt{\sec^2 x - 1}}$ | $\pm\sqrt{\csc^2 x -}$ |
| $\sec x =$ | $\dfrac{\pm 1}{\sqrt{1-\sin^2 x}}$ | $\dfrac{1}{\cos x}$ | $\pm\sqrt{1+\tan^2 x}$ | $\dfrac{\pm\sqrt{1+\cot^2 x}}{\cot x}$ | $\sec x$ | $\dfrac{\pm\csc x}{\sqrt{\csc^2 x - 1}}$ |
| $\csc x =$ | $\dfrac{1}{\sin x}$ | $\dfrac{\pm 1}{\sqrt{1-\cos^2 x}}$ | $\dfrac{\pm\sqrt{1+\tan^2 x}}{\tan x}$ | $\pm\sqrt{1+\cot^2 x}$ | $\dfrac{\pm\sec x}{\sqrt{\sec^2 x - 1}}$ | $\csc x$ |