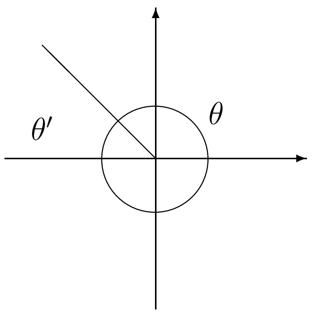
Math 002 — Related (Reference) Angle (θ')

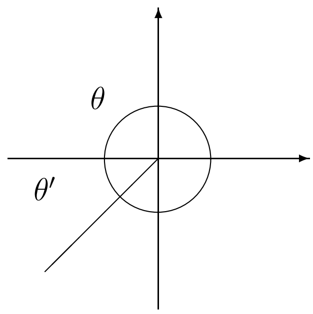
We like to find the **trigonometric functions** of large angles such as $240°$, $330°$ by using smaller angles such as $30°$, $45°$. First let us have this definition :

<u>Definition</u> : The related angle θ' of any angle θ in standard position is the **<u>acute</u>** angle between the **terminal** side of the angle θ and the **x-axis** . So $0 < \theta' < \frac{\pi}{2}$.

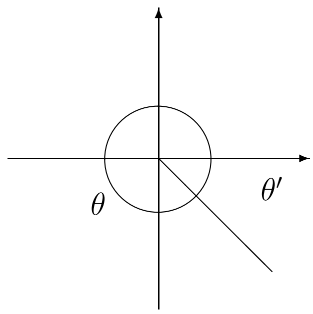
We are excluding the **quadrantal** angles such as $180°$, $270°$ because it's easy to find the value of the trigonometric functions of these angles .



θ lies in quad. II
$\theta' = 180° - \theta$



θ lies in quad. III
$\theta' = \theta - 180°$



θ lies in quad. IV
$\theta' = 360° - \theta$

<u>Example</u> : Let us find the related angle for each of the following angles :

a) If $\theta = 150°$, then $\theta' = 180° - 150° = 30°$
b) If $\theta = 240°$, then $\theta' = 240° - 180° = 60°$
c) If $\theta = 225°$, then $\theta' = 225° - 180° = 45°$
d) If $\theta = 330°$, then $\theta' = 360° - 330° = 30°$

Notice that we are using $180°$ and $360°$ in finding the related angle θ' but not $90°$ and $270°$.

Before we go to the next point we have to remember two things :
i) The value of all trigonometric functions of θ' is **<u>positive</u>** because θ' is an **<u>acute</u>** angle .
ii) We should know the value of the trigonometric functions of the famous angles $30°$, $45°$ and $60°$ by using the $30° - 60°$ triangle and $45° - 45°$ triangle .

<u>Rule</u> : It can be shown by using equivalent triangles that for any angle θ in standard position we have $\sin\theta = \pm\sin\theta'$, $\cos\theta = \pm\cos\theta'$, $\tan\theta = \pm\tan\theta'$, $\csc\theta = \pm\csc\theta'$ etc ...

Notice that $\pm$ depends on the position of the angle θ , e.g. , if θ lies in quadrant II , then $\sin\theta = \sin\theta'$ and $\csc\theta = \csc\theta'$ but $\cos\theta = -\cos\theta'$, $\sec\theta = -\sec\theta'$, $\tan\theta = -\tan\theta'$ and $\cot\theta = -\cot\theta'$.

<u>Example</u> : Let us find the value of the following :

$\cos 150° = -\cos(180° - 150°) = -\cos 30° = -\frac{\sqrt{3}}{2}$; $\tan 225° = \tan(225° - 180°) = \tan 45° = 1$
$\sin 210° = -\sin(210° - 180°) = -\sin 30° = -\frac{1}{2}$; $\csc 135° = \csc(180° - 135°) = \csc 45° = \sqrt{2}$
$\cot 330° = -\cot(360° - 330°) = -\cot 30° = -\sqrt{3}$; $\sec 300° = \sec(360° - 300°) = \sec 60° = 2$
$\tan 675° = \tan(3 \cdot 180° + 135°) = \tan 135° = -\tan(180° - 135°) = -\tan 45° = -1$
$\csc(-2010°) = -\csc(4 \cdot 360° + 210°) = -\csc 210° = -[-\csc(210° - 180°)] = \csc 30° = 2$

BY : A.AL-SHALLALI