Math 002 — Common Mistakes — Exp. & Log. Functions

⋆ Let us agree that when we write $\log_a x$, then $x > 0$, $a > 0$ and $a \neq 1$.

1. In general $-a^n \neq (-a)^n$, e . g . , $-3^2 = -9 \neq (-3)^2 = 9$.

2. $a \cdot b^x \neq (ab)^x$, e . g . , $3 \cdot 5^2 \neq (15)^2 = 225$ but $3 \cdot 5^2 = 3(25) = 75$.

3. $a^{n^m} \neq (a^n)^m = a^{mn}$, e . g . , $a^{3^2} = a^9 \neq (a^3)^2 = a^6$.

4. In general $x^n = y^n \not\Longrightarrow x = y$, in fact :

   (a) If n is an <u>even</u> integer then $x^n = y^n \Longrightarrow x = \pm y$.

   (b) If n is an <u>odd</u> integer then $x^n = y^n \Longrightarrow x = y$ only .

5. $\ln 10 \neq 1$ and $\log e \neq 1$ but $\ln e = 1$, $\log 10 = 1$ and $(\ln 10) \cdot (\log e) = 1$, because $(\log_a b) \cdot (\log_b a) = 1$.

6. $\log_a(x+y) \neq \log_a x + \log_a y$ but $\log_a(x \cdot y) = \log_a x + \log_a y$. There is **no rule** for $\log_a(x+y)$ or $\log_a(x-y)$.

7. If x and y are negative , then xy is positive , but $\log_a(x \cdot y) \neq \log_a x + \log_a y$. This is because $\log_a x$ and $\log_a y$ are <u>not</u> defined .

8. For $x > 0$, $\log_a x^n \neq (\log_a x)^n$ but $\log_a x^n = n \log_a x$, e . g . , if $\log 2 = 0.3$, then $\log 4 \neq (\log 2)^2 = 0.09$ but $\log 4 = 2 \log 2 = 2(0.3) = 0.6$.

9. For $x \neq 0$, $\log_a x^2 \neq 2 \log_a x$, e . g . , $\log_a(-3)^2 \neq 2 \log_a(-3)$ but $\log_a x^2 = 2 \log_a |x|$. In general $\log_a x^n = n \log_a |x|$, where n is an <u>even</u> integer .

10. $\log_a(x^n \cdot y) \neq n \log_a(x \cdot y)$ but $\log_a(x^n \cdot y) = n \log_a x + \log_a y$ and $\log_a(x \cdot y)^n = n \log_a(x \cdot y) = n \log_a x + n \log_a y$.

11. $\dfrac{\log_a x}{\log_a y} \neq \log_a x - \log_a y$ but

    (a) $\dfrac{\log_a x}{\log_a y} = \log_y x$, where $y \neq 1$ and

    (b) $\log_a x - \log_a y = \log_a \dfrac{x}{y}$.

12. $e^{\log x} \neq x$ and $10^{\ln x} \neq x$ but $10^{\log x} = x$ and $e^{\ln x} = x$.

13. $a^{n \log_a x} \neq nx$ but $a^{n \log_a x} = a^{\log_a x^n} = x^n$, e . g . , $10^{3 \log 2} = 8$ and $e^{-2 \ln 3} = \dfrac{1}{9}$.

14. $a^{(x+y)} \neq a^x + a^y$ but $a^{(x+y)} = a^x \cdot a^y$, e . g . , $e^{(3+2\ln 5)} = e^3 \cdot e^{2 \ln 5} = 25\, e^3$.

15. $\log_{a^n} x \neq n \log_a x$ but $\log_{a^n} x = \dfrac{\log_a x}{\log_a a^n} = \dfrac{\log_a x}{n} = \dfrac{1}{n} \log_a x$, in particular $\log_{\frac{1}{a}} x = \dfrac{1}{-1} \log_a x = -\log_a x$. Moreover $\log_{a^n} x^m = \dfrac{m}{n} \log_a x$, e . g . , $\log_{\frac{1}{2}} 8 = -\log_2 2^3 = -3\log_2 2 = -3$; $\log_{e^3} e^4 = \frac{4}{3} \ln e = \frac{4}{3}$; $\log_{16} \sqrt{32} = \dfrac{5/2}{4} \log_2 2 = \frac{5}{8}$.

16. $\log_a x + \log_a y = \log_a z \;\not\Longrightarrow\; x + y = z$ but

    (a) $\log_a x + \log_a y = \log_a z \implies xy = z$.

    (b) $\log_a x - \log_a y = \log_a z \implies \frac{x}{y} = z$.

    (c) $\log_a x + \log_a y = k \implies xy = a^k$ etc .

    So we have to write each side of a log. equation as a single logarithm , e . g . , $\ln x - \ln y + 2\ln z = 5 \implies \ln x + \ln y^{-1} + \ln z^2 = \ln e^5 \implies \ln(xy^{-1}z^2) = \ln e^5 \implies \dfrac{xz^2}{y} = e^5$.

17. In general $\log_a x < \log_a y \;\not\Longrightarrow\; x < y$, in fact :

    (a) For $0 < a < 1$: $\log_a x < \log_a y \implies x > y > 0$.

    (b) For $a > 1$: $\log_a x < \log_a y \implies 0 < x < y$.

    The reason for this is that $y = \log_a x$ is a decreasing function if $0 < a < 1$ and an increasing function if $a > 1$. For example , $\log_3(x-1) < 4 \implies \log_3(x-1) < 4\log_3 3 = \log_3 81 \implies 0 < x - 1 < 81 \implies 1 < x < 82$, then S . S . $= (1, 82)$. But $\log_{\frac{1}{2}}(x+4) < 3 \implies \log_{\frac{1}{2}}(x+4) < 3\log_{\frac{1}{2}} \frac{1}{2} = \log_{\frac{1}{2}} \frac{1}{8} \implies x + 4 > \frac{1}{8} \implies x > -\frac{31}{8}$ (because $0 < a = \frac{1}{2} < 1$) , then S . S . $= (-\frac{31}{8}, \infty)$.

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