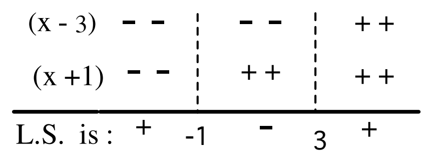
Math 001.10 **Quiz # 3** **Sem. I , Dec 16 , 2006**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. If $f(x)=\sqrt{4-x^2}$ and $g(x)=\sqrt{x^2-2x-3}$, then find the domain of $(f+g)(x)$ and $\left(\frac{f}{g}\right)(x)$.



<u>**Ans.**</u> $\mathrm{D_f}:\ 4-x^2\geq 0 \Longrightarrow x^2\leq 4 \Longrightarrow |x|\leq 2$

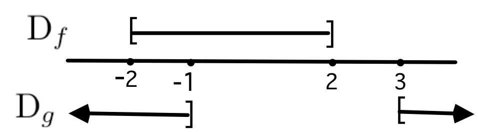
$\Longrightarrow -2\leq x\leq 2 \Longrightarrow \mathrm{D_f}=[-2,2]$

$\mathrm{D_g}:\ x^2-2x-3\geq 0 \Longrightarrow (x-3)(x+1)\geq 0 \Longrightarrow$

(by sign test) $\mathrm{D_g}=(-\infty,-1]\cup[3,\infty)$

$\mathrm{D_{(f+g)}}=\mathrm{D_f}\cap\mathrm{D_g}=[-2,2]\cap\Big((-\infty,-1]\cup[3,\infty)\Big)$

$=[-2,-1]$



$\mathrm{D}_{\frac{f}{g}}=\mathrm{D_f}\cap\mathrm{D_g}$ and $g(x)\neq 0 \Longrightarrow \mathrm{D}_{\frac{f}{g}}=[-2,-1]$ and

$x\neq 3\ ,\ -1 \Longrightarrow \mathrm{D}_{\frac{f}{g}}=[-2,-1)$

2. Use synthetic division to find the quotient and the remainder of dividing $p(x)=2x^4+3x^3-5x+2$ by $x+2$. Is $(x+2)$ a factor of $p(x)$? Why?

<u>**Ans.**</u> Quotient $=q(x)=2x^3-x^2+2x-9$, remainder=r $=20$.

Since $\mathrm{r}=20\neq 0$, then $x+2$ is not a factor of $p(x)$.

	2	3	0	- 5	2
- 2		- 4	2	- 4	18
	2	-1	2	- 9	20

3. If $y=f(x)=\dfrac{1-x}{2x+3}$, then find $f^{-1}(x)$ and its domain and range.

<u>**Ans.**</u> $f(x)$ is 1–1, then $f^{-1}(x)$ exists. Interchange x with y: $x=\dfrac{1-y}{2y+3}$. Next solve for y:

$2xy+3x=1-y \Longrightarrow 2xy+y=1-3x \Longrightarrow (2x+1)y=1-3x \Longrightarrow y=f^{-1}(x)=\dfrac{1-3x}{1+2x}$

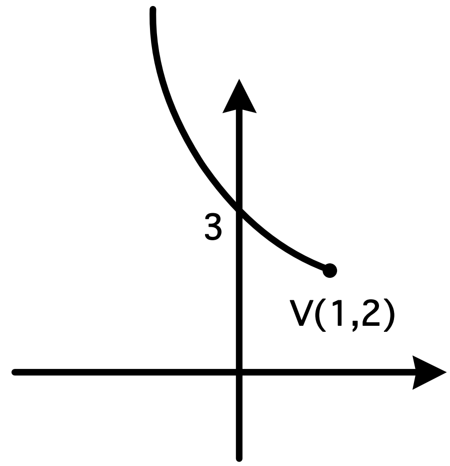
$\mathrm{D}_{f^{-1}}=\Re-\left\{-\dfrac{1}{2}\right\}=(-\infty,-\dfrac{1}{2})\cup(-\dfrac{1}{2},\infty)$

$\mathrm{R}_{f^{-1}}=\mathrm{D}_f=\Re-\left\{-\dfrac{3}{2}\right\}=(-\infty,-\dfrac{3}{2})\cup(-\dfrac{3}{2},\infty)$

Math 001.26 **Quiz # 3** **Sem. I , Dec 16 , 2006**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Let $y = f(x) = x^2 - 2x + 3$, $x \le 1$. Find $f^{-1}(x)$ and its domain and range.



<u>Ans.</u> $y = (x^2 - 2x + 1) + 2 = (x-1)^2 + 2$, $x \le 1$. This is the left half of a parabola, opens upwards, $\mathrm{V}(1,2)$. From the graph $f(x)$ is a 1–1 function, so $f^{-1}(x)$ exists. Interchange x with y: $x = (y-1)^2 + 2$. Next solve for y: $(y-1)^2 = x-2 \implies y-1 = \pm\sqrt{x-2} \implies y = 1 \pm \sqrt{x-2}$, but $y \le 1 \implies y = f^{-1}(x) = 1 - \sqrt{x-2}$.
$\mathrm{D}_{f^{-1}} = \mathrm{R}_f = [2, \infty)$ and $\mathrm{R}_{f^{-1}} = \mathrm{D}_f = (-\infty, 1]$

2. Use synthetic division to find the quotient and the remainder of dividing $p(x) = 2x^5 + 3x^4 - 2x^2 + 5x + +6$ by $x+1$. Is $(x+1)$ a factor of $p(x)$? Why?

<u>Ans.</u> Quotient $= q(x) = 2x^4 + x^3 - x^2 - x + 6$,

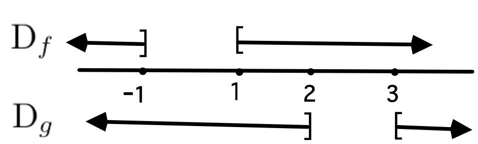
remainder = r = 0. Since r = 0 , then $x + 1$ is a factor of $p(x)$.

	2	3	0	- 2	5	6
- 1		- 2	-1	1	1	- 6
	2	1	-1	-1	6	0

3. If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = \sqrt{x^2 - 5x + 6}$, then find the domain of $(f-g)(x)$ and $\left(\frac{g}{f}\right)(x)$.

<u>Ans.</u> D_f : $x^2 - 1 \ge 0 \implies x^2 \ge 1 \implies |x| \ge 1 \implies$
$x \le -1$ or $x \ge 1 \implies \mathrm{D}_f = (-\infty, -1] \cup [1, \infty)$
D_g : $x^2 - 5x + 6 \ge 0 \implies (x-2)(x-3) \ge 0 \implies$
(by sign test) $\mathrm{D}_g = (-\infty, 2] \cup [3, \infty)$
$\mathrm{D}_{(f-g)} = \mathrm{D}_f \cap \mathrm{D}_g = (-\infty, -1] \cup [1, 2] \cup [3, \infty)$
$\mathrm{D}_{\frac{f}{g}} = \mathrm{D}_f \cap \mathrm{D}_g$ and $g(x) \ne 0 \implies$
$\mathrm{D}_{\frac{f}{g}} = \mathrm{D}_f \cap \mathrm{D}_g$ and $x \ne 2\,,\,3 \implies$
$\mathrm{D}_{\frac{f}{g}} = (-\infty, -1] \cup [1, 2) \cup (3, \infty)$

(x - 2)	- -		+ +		+ +
(x -3)	- -		- -		+ +
L.S. is :	+	2	-	3	+



Math 001.34 **Quiz # 3** **Sem. I , Dec 16 , 2006**

Name : ________________ **ID. # :** ______ **SER. # :** ____

1. Find the remainder of dividing $p(x) = -2x^{101} + 3x^{43} - 5$ by $x + i$.

<u>**Ans.**</u> The remainder $= p(c) = p(-i) = -2(-i)^{101} + 3(-i)^{43} - 5 = 2i^{101} - 3i^{43} - 5 = 2i - 3i^3 - 5$

$= 2i + 3i - 5 = -5 + 5i$

2. If $f(x) = 2 - 3x$ and $(f \circ g)(x) = 4x + 3$, then find (i) $g(x)$ (ii) the value of $(g \circ f)(-2)$

<u>**Ans.**</u> (i) $4x + 3 = (f \circ g)(x) = f\Big(g(x)\Big) = 2 - 3g(x) \implies 3g(x) = 2 - 4x - 3 = -1 - 4x$

$. \implies g(x) = -\dfrac{1}{3} - \dfrac{4}{3}x$

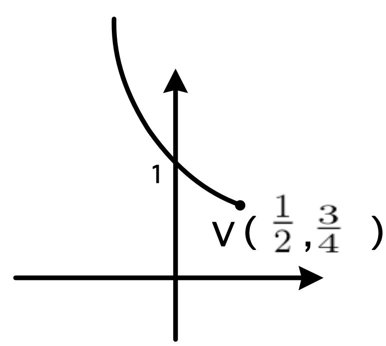
(ii) $(g \circ f)(-2) = g\Big(f(-2)\Big) = g(2+6) = g(8) = -\dfrac{1}{3} - \dfrac{4}{3}(8) = -\dfrac{1}{3} - \dfrac{32}{3} = -\dfrac{33}{3} = -11$

3. Let $y = f(x) = x^2 - x + 1$, $x \leq \dfrac{1}{2}$. Find $f^{-1}(x)$ and its domain and range.

<u>**Ans.**</u> $y = (x^2 - x + \dfrac{1}{4}) + \dfrac{3}{4} = (x - \dfrac{1}{2})^2 + \dfrac{3}{4}$, $x \leq \dfrac{1}{2}$.

This is the left half of a parabola, opens upwards, $\mathrm{V}(\dfrac{1}{2}, \dfrac{3}{4})$.

From the graph $f(x)$ is a 1–1 function, so $f^{-1}(x)$ exists.



Interchange x with y: $x = (y - \dfrac{1}{2})^2 + \dfrac{3}{4}$.

Next solve for y: $(y - \dfrac{1}{2})^2 = x - \dfrac{3}{4} \implies$

$y - \dfrac{1}{2} = \pm\sqrt{x - \dfrac{3}{4}} \implies y = \dfrac{1}{2} \pm \sqrt{x - \dfrac{3}{4}}$,

but $y \leq \dfrac{1}{2} \implies y = f^{-1}(x) = \dfrac{1}{2} - \sqrt{x - \dfrac{3}{4}}$.

$\mathrm{D}_{f^{-1}} = \mathrm{R}_f = [\dfrac{3}{4}, \infty)$ and $\mathrm{R}_{f^{-1}} = \mathrm{D}_f = (-\infty, \dfrac{1}{2}]$