Math 001.10 **Quiz # 3** **Sem. I , Dec 16 , 2006**

Name : ______________ **ID. # :** ______ **SER. # :** ____

1. If $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{x^2 - 2x - 3}$, then find the domain of $(f+g)(x)$ and $\left(\frac{f}{g}\right)(x)$.

<u>Ans.</u> $\mathrm{D_f} : 4 - x^2 \geq 0 \implies x^2 \leq 4 \implies |x| \leq 2$

$\implies -2 \leq x \leq 2 \implies \mathrm{D_f} = [-2, 2]$

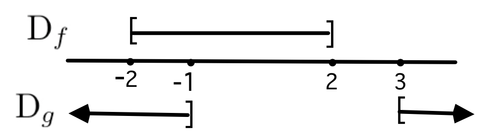
$\mathrm{D_g} : x^2 - 2x - 3 \geq 0 \implies (x-3)(x+1) \geq 0 \implies$

(by sign test) $\mathrm{D_g} = (-\infty, -1] \cup [3, \infty)$

		-1		3	
(x - 3)	- -		- -		+ +
(x +1)	- -		+ +		+ +
L.S. is :	+	-1	-	3	+

$\mathrm{D_{(f+g)}} = \mathrm{D_f} \cap \mathrm{D_g} = [-2, 2] \cap \Big((-\infty, -1] \cup [3, \infty)\Big)$

$= [-2, -1]$



$\mathrm{D}_{\frac{f}{g}} = \mathrm{D_f} \cap \mathrm{D_g}$ and $g(x) \neq 0 \implies \mathrm{D}_{\frac{f}{g}} = [-2, -1]$ and

$x \neq 3\ ,\ -1 \implies \mathrm{D}_{\frac{f}{g}} = [-2, -1)$

2. Use synthetic division to find the quotient and the remainder of dividing $p(x) = 2x^4 + 3x^3 - 5x + 2$ by $x + 2$. Is $(x + 2)$ a factor of $p(x)$? Why?

<u>Ans.</u> Quotient $= q(x) = 2x^3 - x^2 + 2x - 9$, remainder=r $= 20$.

Since $\mathrm{r} = 20 \neq 0$, then $x + 2$ is not a factor of $p(x)$.

$$\begin{array}{r|rrrrr} & 2 & 3 & 0 & -5 & 2 \\ -2 & & -4 & 2 & -4 & 18 \\ \hline & 2 & -1 & 2 & -9 & 20 \end{array}$$

3. If $y = f(x) = \dfrac{1 - x}{2x + 3}$, then find $f^{-1}(x)$ and its domain and range.

<u>Ans.</u> $f(x)$ is 1–1, then $f^{-1}(x)$ exists. Interchange x with y: $x = \dfrac{1 - y}{2y + 3}$. Next solve for y:

$2xy + 3x = 1 - y \implies 2xy + y = 1 - 3x \implies (2x + 1)y = 1 - 3x \implies y = f^{-1}(x) = \dfrac{1 - 3x}{1 + 2x}$

$\mathrm{D}_{f^{-1}} = \Re - \left\{-\frac{1}{2}\right\} = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

$\mathrm{R}_{f^{-1}} = \mathrm{D}_f = \Re - \left\{-\frac{3}{2}\right\} = (-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

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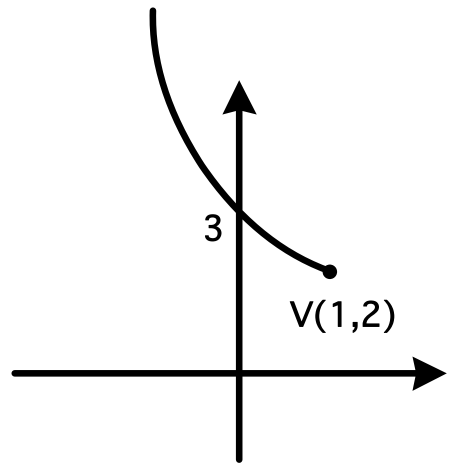
1. Let $y = f(x) = x^2 - 2x + 3$, $x \leq 1$. Find $f^{-1}(x)$ and its domain and range.



<u>Ans.</u> $y = (x^2 - 2x + 1) + 2 = (x - 1)^2 + 2$, $x \leq 1$. This is the left half of a parabola, opens upwards, $\mathrm{V}(1, 2)$. From the graph $f(x)$ is a 1–1 function, so $f^{-1}(x)$ exists. Interchange x with y: $x = (y - 1)^2 + 2$. Next solve for y: $(y-1)^2 = x-2 \implies y-1 = \pm\sqrt{x-2} \implies y = 1 \pm \sqrt{x-2}$, but $y \leq 1 \implies y = f^{-1}(x) = 1 - \sqrt{x-2}$.
$\mathrm{D}_{f^{-1}} = \mathrm{R}_f = [2, \infty)$ and $\mathrm{R}_{f^{-1}} = \mathrm{D}_f = (-\infty, 1]$

2. Use synthetic division to find the quotient and the remainder of dividing $p(x) = 2x^5 + 3x^4 - 2x^2 + 5x + +6$ by $x + 1$. Is $(x + 1)$ a factor of $p(x)$? Why?

<u>Ans.</u> Quotient $= q(x) = 2x^4 + x^3 - x^2 - x + 6$,

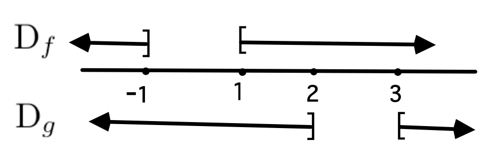
remainder $= \mathrm{r} = 0$. Since $\mathrm{r} = 0$, then $x + 1$ is a factor of $p(x)$.

	2	3	0	- 2	5	6
- 1		- 2	-1	1	1	- 6
	2	1	-1	-1	6	0

3. If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = \sqrt{x^2 - 5x + 6}$, then find the domain of $(f - g)(x)$ and $\left(\frac{g}{f}\right)(x)$.

<u>Ans.</u> D_f : $x^2 - 1 \geq 0 \implies x^2 \geq 1 \implies |x| \geq 1 \implies$
$x \leq -1$ or $x \geq 1 \implies \mathrm{D}_f = (-\infty, -1] \cup [1, \infty)$
D_g : $x^2 - 5x + 6 \geq 0 \implies (x - 2)(x - 3) \geq 0 \implies$
(by sign test) $\mathrm{D}_g = (-\infty, 2] \cup [3, \infty)$
$\mathrm{D}_{(f-g)} = \mathrm{D}_f \cap \mathrm{D}_g = (-\infty, -1] \cup [1, 2] \cup [3, \infty)$
$\mathrm{D}_{\frac{f}{g}} = \mathrm{D}_f \cap \mathrm{D}_g$ and $g(x) \neq 0 \implies$
$\mathrm{D}_{\frac{f}{g}} = \mathrm{D}_f \cap \mathrm{D}_g$ and $x \neq 2$, $3 \implies$
$\mathrm{D}_{\frac{f}{g}} = (-\infty, -1] \cup [1, 2) \cup (3, \infty)$

(x - 2)	- -		+ +		+ +
(x -3)	- -		- -		+ +
L.S. is :	+	2	-	3	+



Math 001.34 **Quiz # 3** **Sem. I , Dec 16 , 2006**

Name : ______________________________ **ID. # :** ______ **SER. # :** ____

1. Find the remainder of dividing $p(x) = -2x^{101} + 3x^{43} - 5$ by $x + i$.

Ans. The remainder $= p(c) = p(-i) = -2(-i)^{101} + 3(-i)^{43} - 5 = 2i^{101} - 3i^{43} - 5 = 2i - 3i^{3} - 5$

$= 2i + 3i - 5 = -5 + 5i$

2. If $f(x) = 2 - 3x$ and $(f \circ g)(x) = 4x + 3$, then find (i) $g(x)$ (ii) the value of $(g \circ f)(-2)$

Ans. (i) $4x + 3 = (f \circ g)(x) = f\Big(g(x)\Big) = 2 - 3g(x) \implies 3g(x) = 2 - 4x - 3 = -1 - 4x$

. $\implies g(x) = -\dfrac{1}{3} - \dfrac{4}{3}x$

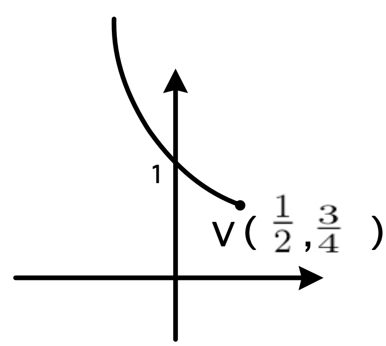
(ii) $(g \circ f)(-2) = g\Big(f(-2)\Big) = g(2+6) = g(8) = -\dfrac{1}{3} - \dfrac{4}{3}(8) = -\dfrac{1}{3} - \dfrac{32}{3} = -\dfrac{33}{3} = -11$

3. Let $y = f(x) = x^2 - x + 1$, $x \le \dfrac{1}{2}$. Find $f^{-1}(x)$ and its domain and range.

Ans. $y = (x^2 - x + \dfrac{1}{4}) + \dfrac{3}{4} = (x - \dfrac{1}{2})^2 + \dfrac{3}{4}$, $x \le \dfrac{1}{2}$.

This is the left half of a parabola, opens upwards, $\mathrm{V}(\dfrac{1}{2}, \dfrac{3}{4})$.

From the graph $f(x)$ is a 1–1 function, so $f^{-1}(x)$ exists.



Interchange x with y: $x = (y - \dfrac{1}{2})^2 + \dfrac{3}{4}$.

Next solve for y: $(y - \dfrac{1}{2})^2 = x - \dfrac{3}{4} \implies$

$y - \dfrac{1}{2} = \pm\sqrt{x - \dfrac{3}{4}} \implies y = \dfrac{1}{2} \pm \sqrt{x - \dfrac{3}{4}}$,

but $y \le \dfrac{1}{2} \implies y = f^{-1}(x) = \dfrac{1}{2} - \sqrt{x - \dfrac{3}{4}}$.

$\mathrm{D}_{f^{-1}} = \mathrm{R}_f = [\dfrac{3}{4}, \infty)$ and $\mathrm{R}_{f^{-1}} = \mathrm{D}_f = (-\infty, \dfrac{1}{2}]$