Math 001 Solution of Midterm Exam Semester I, Term 061

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1. $\dfrac{2+(x-2)^{-1}}{3-2(2-x)^{-1}}$ simlifies to

(a) $\dfrac{2x+3}{3x-4}$ (b) $\dfrac{2x+5}{-3x-4}$ (c) $\dfrac{2x-5}{-3x+4}$ (d) $\dfrac{3}{5}$ ✓(e) $\dfrac{2x-3}{3x-4}$

Ans: $= \dfrac{2+(x-2)^{-1}}{3+2(x-2)^{-1}} \cdot \dfrac{x-2}{x-2} = \dfrac{2(x-2)+1}{3(x-2)+2} = \dfrac{2x-3}{3x-4}$

2. If $i=\sqrt{-1}$ and $z_1 = \dfrac{8-i}{2+3i}$, and $z_2 = i^{-83}$, then $z_1 + z_2 =$

(a) $1-4i$ ✓(b) $1-i$ (c) $-\dfrac{13}{5}+\dfrac{31}{5}i$ (d) $1+3i$ (e) $-1+3i$

Ans: $z_1 = \dfrac{8-i}{2+3i} \cdot \dfrac{2-3i}{2-3i} = \dfrac{(16-3)+(-2-24)i}{4+9} = \dfrac{13}{13} - \dfrac{26}{13}i = 1-2i$ and

$z_2 = \dfrac{1}{i^{83}} \cdot \dfrac{i}{i} = \dfrac{i}{i^{84}} = \dfrac{i}{1} = i$. Thus $z_1 + z_2 = 1-2i+i = 1-i$

3. The solution set of the equation $\sqrt{x+9}-3 = x$ contains

✓(a) one rational number (b) one negative rational number (c) no real numbers (d) two rational numbers (e) one positive rational number

Ans: $\sqrt{x+9} = x+3 \implies x+9 = x^2+6x+9 \implies x^2+5x=0 \implies x(x+5)=0 \implies x=0$ or $x=-5$. Checking, for $x=0:\ \sqrt{9}-3 \overset{?}{=} 0$ (yes). For $x=-5:\ \sqrt{4}-3 \overset{?}{=} -5$ (no).

Therefore S.S. $=\{0\}$, where 0 is a rational number

4. One of the factors of $5x^3y - 5xy^3 + 6x^2y - 6xy^2$ is

✓(a) $5x+5y+6$ (b) $5x+5y-6$ (c) $5x-5y+6$ (d) $5x+5y+11$ (e) $5x-5y-6$

Ans: $(5x^3y - 5xy^3) + (6x^2y - 6xy^2) = 5xy(x^2-y^2) + 6xy(x-y) = 5xy(x-y)(x+y) + 6xy(x-y)$ $= xy(x-y)(5x+5y+6)$. So $5x+5y+6$ is one of the factors.

5. The value of k for which the quadratic equation $kx^2 + 3kx + (2k+1) = 0$ has two equal real solutions is

(a) 1 (b) -4 (c) 0 (d) 2 ✓(e) 4

Ans: $\implies b^2 - 4ac = 0 \implies 9k^2 - 4(k)(2k+1) = 0 \implies 9k^2 - 8k^2 - 4k = 0$

$\implies k^2 - 4k = 0 \implies k(k-4) = 0 \implies k = 0$ or $k = 4$, but $k \neq 0 \implies k = 4$ only

6. The solution set, in interval notation, of the inequality $\dfrac{2x-3}{x^2-36} \leq \dfrac{1}{x+6}$ is

(a) $(-\infty, -3]$ (b) $(-6, -3] \cup (6, \infty)$ (c) $(-\infty, -6] \cup [-3, \infty)$

(d) $(-2, 2)$ ✓(e) $(-\infty, -6] \cup [-3, 6)$

Ans: $\dfrac{2x-3}{(x-6)(x+6)} - \dfrac{1}{x+6} \leq 0$, $x \neq -6$, 6

$\implies \dfrac{2x-3-x+6}{(x-6)(x+6)} \leq 0 \implies \dfrac{x+3}{(x-6)(x+6)} \leq 0.$

By sign test S.S. $= (-\infty, -6) \cup [-3, 6)$

(x+3)	− −		− −		+ +		+ +
(x-6)	− −		− −		− −		+ +
(x+6)	− −		+ +		+ +		+ +
L.S. is :	−	-6	+	-3	−	6	+

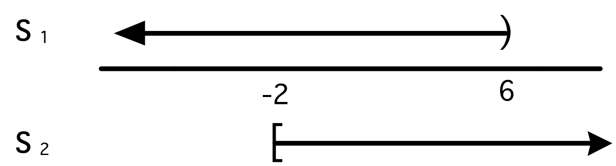
7. The solution set, in interval notation, of the compound inequality $2x - 4 < 8$ and $-2x + 1 \leq 5$ is

(a) $(-\infty, 6)$ ✓(b) $[-2, 6)$ (c) $(-\infty, -2] \cup (6, \infty)$

(d) $[-2, \infty)$ (e) $(6, \infty)$

Ans: $2x < 12$ and $-2x \leq 4 \implies x < 6$ and $x \geq -2$.

So, S.S. $= S_1 \cap S_2 = [-2, 6)$



8. The length L of a rectangle is 2 units less than twice the width W of the rectangle. If the perimeter of the rectangle is 110 units, then L − W is equal to

(a) 53 ✓(b) 17 (c) 50 (d) 19 (e) 16

Ans: $L = 2W - 2$. Perimeter $= 110 = 2L + 2W \implies L + W = 55 \implies (2W - 2) + W = 55$

$\implies 3W = 57 \implies W = 19$ and $L = 2W - 2 = 38 - 2 = 36 \implies L - W = 36 - 19 = 17$ units

9. The solutions of the equation $\frac{1}{2}x^2 + \frac{4}{3}x + 1 = 0$ are

(a) $-\frac{4}{3} \pm \frac{\sqrt{5}}{3}i$ ✓(b) $-\frac{4}{3} \pm \frac{\sqrt{2}}{3}i$ (c) $-\frac{2}{3} \pm \frac{\sqrt{2}}{6}i$ (d) $-\frac{4}{3} \pm \frac{\sqrt{3}}{3}i$ (e) $\frac{4}{3} \pm \frac{2\sqrt{2}}{3}i$

Ans: For the equation $3x^2 + 8x + 6 = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-8 \pm \sqrt{64 - 4(3)(6)}}{6}$

$= \dfrac{-8 \pm \sqrt{64 - 72}}{6} = \dfrac{-8 \pm \sqrt{-8}}{6} = \dfrac{-8 \pm 2\sqrt{2}\, i}{6} = -\dfrac{4}{3} \pm \dfrac{\sqrt{2}}{3}i$

10. The value of the expression $-17 + 3\Big[8x - 4(3x - 2)\Big]$ when $x = -\frac{3}{4}$ is

(a) 22 (b) -5 (c) -11 (d) 13 ✓(e) 16

Ans: The expression $= -17 + 3(8x - 12x + 8) = -17 + 3(-4x + 8) = -17 - 12x + 24 = 7 - 12x$.

The value $= 7 - 12\left(-\frac{3}{4}\right) = 7 + 9 = 16$

11. IF A is the leading coefficient and B is the constant term of the polynomial $(4x - 1)^2 - (2x - 3)^2$, then $\mathrm{A} + \mathrm{B} =$

(a) -6 ✓(b) 4 (c) 5 (d) 20 (e) 16

Ans: $= (16x^2 - 8x + 1) - (4x^2 - 12x + 9) = 12x^2 + 4x - 8 \implies \mathrm{A} = 12$, $\mathrm{B} = -8$, and then

$\mathrm{A} + \mathrm{B} = 12 + (-8) = 4$

12. The expression $\dfrac{y}{y^2 - 2y - 8} - \dfrac{2}{y^2 - 5y + 4} + \dfrac{1}{y^2 + y - 2}$ simplifies to

✓(a) $\dfrac{1}{y - 1}$ (b) $\dfrac{1}{y + 2}$ (c) $\dfrac{y - 2}{(y + 2)(y - 4)}$ (d) $\dfrac{y - 4}{(y + 4)(y - 1)}$ (e) $\dfrac{y}{y - 4}$

Ans: $= \dfrac{y}{(y - 4)(y + 2)} - \dfrac{2}{(y - 4)(y - 1)} + \dfrac{1}{(y + 2)(y - 1)} = \dfrac{y(y - 1) - 2(y + 2) + (y - 4)}{(y - 4)(y + 2)(y - 1)}$

$= \dfrac{y^2 - y - 2y - 4 + y - 4}{(y - 4)(y + 2)(y - 1)} = \dfrac{y^2 - 2y - 8}{(y - 4)(y + 2)(y - 1)} = \dfrac{\cancel{(y - 4)}\,\cancel{(y + 2)}}{\cancel{(y - 4)}\,\cancel{(y + 2)}\,(y - 1)} = \dfrac{1}{y - 1}$

13. For $x > 0$, and $y > 0$, the expression $\left[\dfrac{(2x^2y)^{-1}(2x^3y^{-2})^2}{2(xy)^{-3}(x^5y^{-2})^{-1}}\right]^{-1/2}$ simplifies to

✓(a) $\dfrac{y^2}{x^6}$ (b) xy^4 (c) x^2y^3 (d) $\dfrac{y}{x^3}$ (e) $\dfrac{x^2}{y^6}$

Ans: $= \left[\dfrac{(2x^3y^{-2})^2(xy)^3x^5y^{-2})}{2(2x^2y)}\right]^{-1/2} = \left[\dfrac{4\ x^6\ y^{-4}\ x^3\ y^3\ x^5\ y^{-2}}{4\ x^2\ y}\right]^{-1/2} = \left[\dfrac{x^{14}y^3}{x^2\ y\ y^6}\right]^{-1/2} = \left[\dfrac{x^{12}}{y^4}\right]^{-1/2}$

$= \dfrac{x^{-6}}{y^{-2}} = \dfrac{y^2}{x^6}$

14. The expression $\dfrac{-1}{x+1} + \dfrac{x^3+64}{x^2} \div \dfrac{x^2-4x+16}{x}$ simplifies to

(a) $\dfrac{(x+2)^2}{x^2(x+1)}$ (b) $\dfrac{x+7}{x(x+1)}$ ✓(c) $\dfrac{(x+2)^2}{x(x+1)}$ (d) $\dfrac{x^2-8x+5}{x(x+1)}$ (e) $\dfrac{x+2}{x^2(x+1)^2}$

Ans: $= \dfrac{-1}{x+1} + \dfrac{(x+4)\ \cancel{(x^2-4x+16)}}{x^{\cancel{2}}} \cdot \dfrac{\cancel{x}}{\cancel{(x^2-4x+4)}} = \dfrac{-1}{x+1} + \dfrac{x+4}{x}$

$= \dfrac{-x+(x+1)(x+4)}{x(x+1)} = \dfrac{-x+x^2+5x+4}{x(x+1)} = \dfrac{x^2+4x+4}{x(x+1)} = \dfrac{(x+2)^2}{x(x+1)}$

15. If $(a+c)x + x^2 = (x+a)^2$, then $x =$

(a) $\dfrac{a^2}{a+c}$ ✓(b) $\dfrac{a^2}{c-a}$ (c) $\dfrac{a^2+2a}{a+c}$ (d) $\dfrac{a^2}{3c-a}$ (e) $\dfrac{a^2}{3a+c}$

Ans: $\implies x^2 + ax + cx = x^2 + 2ax + a^2 \implies ax + cx - 2ax = a^2 \implies cx - ax = a^2$

$\implies (c-a)x = a^2 \implies x = \dfrac{a^2}{c-a}$

16. The possible value(s) of k that makes the trinomial $25x^2 + kxy + 64y^2$ a perfect square is (are)

✓(a) ± 80 (b) 40 (c) -40 (d) ± 160 (e) 13

Ans: It is a perfect square when the middle term $= kxy = \pm 2(5x)(8y) \implies k = \pm 2(5)(8) = \pm 80$

17. The number $\dfrac{(6.9\times10^{29})(7.5\times10^{-14})}{0.023\times10^{16}}$ written in scientific notation is given by

(a) 2.25×10^{-5} (b) 2.25×10^{-2} (c) 22.25×10^{6} ✓(d) 2.25×10^{2} (e) 0.225×10^{4}

Ans: $\dfrac{(\not{69}^{\,3})(75)(10^{28}\times10^{-15})}{\not{23}\times10^{13}} = \dfrac{225\times10^{13}}{10^{13}} = 225 = 2.25\times10^{2}$

18. The expression $5x\sqrt[3]{24x^4}+\dfrac{21x^3}{\sqrt[3]{-9x^2}}$ simplifies to

(a) $-4x^2\sqrt[3]{3x}$ ✓(b) $3x^2\sqrt[3]{3x}$ (c) $11x^2\sqrt[3]{9x^2}$ (d) $-16x^2\sqrt[3]{3x}$ (e) $3x^2\sqrt[3]{9x^2}$

Ans: $=(5x)\sqrt[3]{(8x^3)(3x)}+\dfrac{21x^3}{-\sqrt[3]{3^2x^2}}\cdot\dfrac{\sqrt[3]{3x}}{\sqrt[3]{3x}}=(5x)(2x)\sqrt[3]{3x}-\dfrac{21x^3\sqrt[3]{3x}}{3x}$

$=10x^2\sqrt[3]{3x}-7x^2\sqrt[3]{3x}=(10x^2-7x^2)\sqrt[3]{3x}=3x^2\sqrt[3]{3x}$

19. The sum of the real and imaginary parts of the complex number $\dfrac{\left(\sqrt{-4}\right)\left(\sqrt[3]{-27}-\sqrt{-16}\right)}{(1+i)^2}$ is equal to

(a) $4i$ (b) -1 (c) -2 ✓(d) -7 (e) 1

Ans: $=\dfrac{(2i)(-3-4i)}{1+2i+i^2}=\dfrac{(2i)(-3-4i)}{1+2i-1}=\dfrac{(2i)(-3-4i)}{2i}=-3-4i.$

The real part $=-3$, the imaginary part $=-4$ and the sum of them $=(-3)+(-4)=-7$

20. The sum of all real solutions of the equation $\dfrac{1}{x^6}+\dfrac{9}{x^3}+8=0$ is

✓(a) $-\dfrac{3}{2}$ (b) $-\dfrac{1}{2}$ (c) $-\dfrac{9}{8}$ (d) 9 (e) $\dfrac{1}{2}$

Ans: Multiply both sides by x^6 to get the equation $8x^6+9x^3+1=0$. Let $u=x^3$, so the new equation is $8u^2+9u+1=0 \implies (8u+1)(u+1)=0 \implies u=-\dfrac{1}{8}$ or $u=-1 \implies$ $x^3=-\dfrac{1}{8}$ or $x^3=-1$ (both equations have one real solution and two nonreal complex solutions).

The real solutions are $-\dfrac{1}{2}$ and -1. The sum $=-\dfrac{1}{2}-1=-\dfrac{3}{2}$

21. If the equation $(3x-4)(x+1) = -2$ is written in the form $(x+m)^2 = n$, then $m+n$ is equal to

(a) -1 (b) 1 (c) $\frac{35}{36}$ (d) $-\frac{2}{3}$ ✓(e) $\frac{19}{36}$

<u>Ans:</u> $\Longrightarrow 3x^2 - x - 4 = -2 \Longrightarrow 3x^2 - x - 2 = 0 \Longrightarrow x^2 - \frac{1}{3}x = \frac{2}{3}$.

Add $(\frac{1}{2} \cdot \frac{1}{3})^2 = \frac{1}{36}$ to both sides to get the equation $x^2 - \frac{1}{3}x + \frac{1}{36} = \frac{2}{3} + \frac{1}{36}$

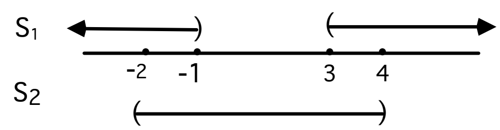
$\Longrightarrow (x - \frac{1}{6})^2 = \frac{25}{36} \Longrightarrow m = -\frac{1}{6}$, $n = \frac{25}{36} \Longrightarrow m + n = -\frac{1}{6} + \frac{25}{36} = \frac{19}{36}$

22. The solution set, in interval notation, of the inequality $2 < |x-1| < 3$ is equal to

(a) $(-2,-1)\cup(3,\infty)$ (b) $(-\infty,-2)\cup(4,\infty)$ (c) $(-2,4)$ (d) $(-1,3)$ ✓(e) $(-2,-1)\cup(3,4)$

<u>Ans:</u> $\Longrightarrow |x-1| > 2$ and $|x-1| < 3 \Longrightarrow$

$\left(x - 1 < -2 \text{ or } x - 1 > 2\right)$. and $-3 < x - 1 < 3$



$\Longrightarrow \left(x < -1 \text{ or } x > 3\right)$ and $-2 < x < 4$

$\Longrightarrow$ S.S. $= S_1 \cap S_2 = (-2,-1) \cup (3,4)$

23. The solution set of the equation $\frac{2|x+2|}{3} - \frac{1}{2} = \frac{4x+5}{6}$ is equal to

(a) $(-\infty,-2]$ (b) $\{-2\}$ ✓(c) $[-2,\infty)$ (d) $(-\infty,\infty)$ (e) ϕ

<u>Ans:</u> Multiply by 6 to get $4|x+2|-3 = 4x+5 \Longrightarrow 4|x+2| = 4x+8 = 4(x+2) \Longrightarrow |x+2| = x+2$ which is always true when $x + 2 \geq 0 \Longrightarrow x \geq -2 \Longrightarrow$ S.S $= [-2,\infty)$

24. Which one of the following equations is **NOT** an **Identity**?

(a) $(x-3)^2 = x^2 - 6x + 9$ (b) $6x - 5 = -3(1-2x) - 2$

(c) $\frac{4}{4x^2+8} = \frac{-2}{-4-2x^2}$ ✓(d) $\frac{x+2}{x+4} = \frac{1}{2}$ (e) $\frac{1}{3}x + 2 = \frac{x+6}{3}$

<u>Ans:</u> In the parts (a) , (b) , (c) , and (e), the left side and the right are identical, so they are identities. In part (d): $\frac{x+2}{x+4} = \frac{1}{2} \Longrightarrow 2x + 4 = x + 4 \Longrightarrow x = 0$, so it is a conditional equation.

25. If the sum and the product of the two roots of the equations $2x^2 + bx + c = 0$ are -4 and $-\frac{3}{2}$ respectively, then $b + c$ is equal to

✓(a) 5 (b) 11 (c) 6 (d) $-\frac{7}{2}$ (e) -6

Ans: Sum of roots $= -\frac{b}{a}$ and the product of roots $= \frac{c}{a}$

$\Longrightarrow -4 = -\frac{b}{2} \Longrightarrow b = 8$ and $-\frac{3}{2} = \frac{c}{2} \Longrightarrow c = -3 \Longrightarrow b + c = 8 - 3 = 5$