Math 001 Solution of Final Exam Semester I, Term 061
Done By: A. Al-Shallali

1. The sum of the solutions of the equation $\dfrac{11|x-5|-10}{2+2|x-5|} = 4$ is equal to

(a) -11 (b) -10 (c) 13 ✔(d) 10 (e) -12

Ans: $11|x-5| - 10 = 8 + 8|x-5| \implies 3|x-5| = 18 \implies |x-5| = 6 \implies x-5 = -6$ or $x-5 = 6 \implies x = -1$ or $x = 11 \implies \text{Sum} = -1 + 11 = 10$

2. Let P be a polynomial in x with leading coefficient 1. If P is with lowest degree and has zeros 3 (of multiplicity 2), $1-i$, and $1+i$, then the coefficient of x^2 in P is equal to

(a) 19 (b) -21 ✔(c) 23 (d) -20 (e) 27

Ans: $\mathrm{P}(x) = (x-3)^2[x-(1-i)][x-(1+i)] = (x^2-6x+9)(x^2-2x+2)$.

The coefficient of x^2 is $2 + 9 + (-2)(-6) = 11 + 12 = 23$

3. The expression $(2x-1)^3 - 2(2x-1)^2$ simplifies to

(a) $8x^3 - 20x^2 + 14x + 1$ (b) $8x^3 - 20x^2 - 2x + 1$ (c) $8x^3 - 20x^2 - 14x - 3$

✔(d) $8x^3 - 20x^2 + 14x - 3$ (e) $8x^3 + 20x^2 + 14x + 3$

Ans: $= 8x^3 - 3(4x^2)(1) + 3(2x)(1) - 1 - 2(4x^2 - 4x + 1) = 8x^3 - 12x^2 + 6x - 1 - 8x^2 + 8x - 2$

$= 8x^3 - 20x^2 + 14x - 3$

4. If the line passing through the points $(3, -k)$ and $(k+1, 2)$ is perpendicular to the line $5x + 6y + 7 = 0$, then $k =$

(a) 21 ✔(b) 22 (c) 26 (d) -13 (e) -21

Ans: $m_1 = \dfrac{\Delta y}{\Delta x} = \dfrac{2+k}{k+1-3} = \dfrac{k+2}{k-2}$ and $m_2 = -\dfrac{5}{6}$. $\mathrm{L}_1 \perp \mathrm{L}_2 \implies m_1 \cdot m_2 = -1$

$$\implies \left(\frac{k+2}{k-2}\right)\left(-\frac{5}{6}\right) = -1 \implies -5k - 10 = -6k + 12 \implies k = 22$$

5. The Zero Location Theorem tells us that the graph of the function $f(x) = x^3 - 5x^2 - x + 7$ has an $x-$intercept in the interval

(a) $[0, 1]$ ✓(b) $[-2, -1]$ (c) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (d) $[-3, -2]$ (e) $\left[0, \frac{1}{3}\right]$

Ans: Since $f(-2) = -19 < 0$ and $f(-1) = 2 > 0$, then there is a real zero ($x-$intercept) in the interval $[-2, -1]$

6. The expression $\dfrac{(a+b)(a^{-1} - b^{-1})}{b^{-2} - a^{-2}}$ simplifies to

✓(a) $-ab$ (b) ab^2 (c) $-\dfrac{1}{ab}$ (d) $a^2 b$ (e) $\dfrac{1}{ab}$

Ans: $= \dfrac{(a+b)(a^{-1} - b^{-1})}{b^{-2} - a^{-2}} \cdot \dfrac{a^2b^2}{a^2b^2} = \dfrac{(a+b)(ab^2 - a^2b)}{a^2 - b^2} = \dfrac{ab\,\cancel{(a+b)}\,\cancel{(b-a)}^{-1}}{\cancel{(a+b)}\,\cancel{(a-b)}} = -ab$

7. If the denominators are rationalized, the expression $\dfrac{5\sqrt{5}}{2+\sqrt{5}} + \dfrac{15}{\sqrt{5}} - 3\sqrt{20}$ simplifies to

✓(a) $25 - 13\sqrt{5}$ (b) 0 (c) $10 - 19\sqrt{5}$ (d) $7\sqrt{5} - 25$ (e) $28 - 16\sqrt{5}$

Ans: $= \dfrac{5\sqrt{5}}{2+\sqrt{5}} \cdot \dfrac{2-\sqrt{5}}{2-\sqrt{5}} + \dfrac{15}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} - 3\sqrt{4(5)} = \dfrac{5\sqrt{5}\,(2-\sqrt{5}\,)}{4-5} + \dfrac{15\sqrt{5}}{5} - 6\sqrt{5}$

$= -10\sqrt{5} + 25 + 3\sqrt{5} - 6\sqrt{5} = 25 - 13\sqrt{5}$

8. The set of real zeros of $P(x) = x^4 - 5x^2 - 2x$ consists of

✓(a) two rational and two irrational numbers (b) four irrational numbers (c) four rational numbers

(d) three rational and one irrational numbers (e) one rational and three irrational numbers

Ans: Let $x^4 - 5x^2 - 2x = 0 \Longrightarrow x(x^3 - 5x - 2) = 0 \Longrightarrow$

$\boxed{x=0}$ or $x^3 - 5x - 2 = 0$. Now $p = \pm 1, \pm 2$ and $q = \pm 1 \Longrightarrow p/q = \pm 1, \pm 2$. By synthetic division -2 is another zero $\Longrightarrow P(x) = x(x+2)(x^2 - 2x - 1)$.

Let $x^2 - 2x - 1 = 0 \Longrightarrow x = \dfrac{2 \pm \sqrt{4+4}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$. Zeros are $0, -2, 1+\sqrt{2}$ and $1-\sqrt{2}$, two rational and two irrational numbers.

	1	0	-5	-2
- 2		-2	4	2
	1	-2	-1	0

9. The remainder of dividing $f(x) = x^{96} - 2x^{97} + 3x^{98} - 2x^{99}$ by x+i is

(a) 2 (b) 0 (c) 1 (d) -1 ✓(e) -2

Ans: $r = f(-i) = (-i)^{96} - 2(-i)^{97} + 3(-i)^{98} - 2(-i)^{99} = i^{96} + 2i^{97} + 3i^{98} + 2i^{99}$

$= 1 + 2i + 3(i^2) + 2(i^3) = 1 + 2i - 3 - 2i = -2$

10. The quotient $q(x)$ of dividing $P(x) = x^2 - 3x^3 + 2x - 1$ by $g(x) = x - 3$ is

✓(a) $q(x) = -3x^2 - 8x - 22$ (b) $q(x) = x^2 - 2$ (c) $q(x) = 3x^2 + 2$

(d) $q(x) = -3x^2 + 8x + 12$ (e) $q(x) = -3x^2 - 8x + 20$

Ans:

3	-3	1	2	-1
		-9	-24	-66
	-3	-8	-22	-67

Therefore $q(x) = -3x^2 - 8x - 22$

11. The solution set, in interval notation, of the inequality $\dfrac{x^2 - x - 8}{x - 2} \le 2$ is equal to

(a) $[-4, 1] \cup (2, \infty)$ (b) $[-\infty, -4] \cup [1, 2)$ (c) $[-1, 2) \cup (2, 4]$

(d) $(-\infty, 2) \cup [4, \infty)$ ✓(e) $(-\infty, -1] \cup (2, 4]$

Ans: $\Longrightarrow \dfrac{x^2 - x - 8}{x - 2} - 2 \le 0$, $x \ne 2 \Longrightarrow$

$\dfrac{x^2 - x - 8 - 2x + 4}{x - 2} \le 0 \Longrightarrow \dfrac{x^2 - 3x - 4}{x - 2} \le 0 \Longrightarrow$

$\dfrac{(x - 4)(x + 1)}{x - 2} \le 0 \Longrightarrow$ S.S. $= (-\infty, -1] \cup (2, 4]$

(x - 4)	- -		- -		- -		+ +
(x + 1)	- -		+ +		+ +		+ +
(x - 2)	- -		- -		+ +		+ +
L.S. is :	-	-1	+	2	-	4	+

12. If $f(x) = -\dfrac{3}{x}$, then $\dfrac{1}{h}[f(2+h) - f(2)]$, is equal to

(a) $-\dfrac{6}{2+h}$ ✓(b) $\dfrac{3}{4+2h}$ (c) $-\dfrac{3}{h^2}$ (d) $\dfrac{5}{4+2h}$ (e) $-\dfrac{3h}{4+2h}$

Ans: $= \dfrac{1}{h}\left[\dfrac{-3}{2+h} + \dfrac{3}{2}\right] = \dfrac{1}{h}\left[\dfrac{-6+6+3h}{2(2+h)}\right] = \dfrac{3h}{2h(2+h)} = \dfrac{3}{2(2+h)} = \dfrac{3}{4+2h}$

13. If $1+i$ is a zero of the polynomial $P(x) = x^4 - 2x^3 + 3x^2 - 2x + 2$, then the set of the **remaining zeros** contains

✓(a) three nonreal numbers

(b) one nonreal and two rational numbers

(c) one nonreal and two irrational numbers

(d) two nonreal and one rational numbers

(e) one rational and two irrrational numbers

<u>**Ans:**</u> Since $P(x)$ has only real coefficients, then $1-i$ is also a zero of $P(x)$. When using synthetic division twice, the last quotient is $q(x) = x^2 + 1 = (x-i)(x+i)$. Thus the remaining zeros are i, $-i$ and $1-i$, where all of them are nonreal.

1+i	1	-2	3	-2	2
		1+i	-2	1 + i	-2
1 - i	1	-1+ i	1	-1 + i	0
		1 - i	0	1 - i	
	1	0	1	0	

14. Which one of the following functions is an **odd** function?

(a) $f(x) = \dfrac{x^2+1}{2-x^3}$

(b) $f(x) = \dfrac{x^2+x}{2-x^2}$

(c) $f(x) = \dfrac{x^3+x}{2-x^3}$

(d) $f(x) = \dfrac{x^2+x}{2-x}$

✓(e) $f(x) = \dfrac{x^3+x,}{2-x^2}$

<u>**Ans:**</u> In part (e): $f(-x) = \dfrac{(-x)^3+(-x)}{2-(-x)^2} = \dfrac{-x^3-x}{2-x^2} = -\dfrac{x^3+x}{2-x^2} = -f(x)$

$\Longrightarrow$ $f(x)$ is an odd function.

15. The number of all possible rational zeros of $P(x) = 4x^5 - 2x^3 + 20x^2 + 6$ is

(a) 10

(b) 22

(c) 20

(d) 12

✓(e) 16

<u>**Ans:**</u> $p = \pm 1, \pm 2, \pm 3, \pm 6$ and $q = \pm 1, \pm 2, \pm 4 \Longrightarrow p/q = \pm 1, \pm\dfrac{1}{2}, \pm\dfrac{1}{4}, \pm 2, \pm 3, \pm\dfrac{3}{2}$

$\pm\dfrac{3}{4}, \pm 6.$ Thus there are 16 possible rational zeros of $P(x)$.

16. Which one of the following statements is **TRUE** about the circle $x^2 + y^2 + 4x - 6y + 9 = 0$?

(a) It is tangent to both the $x-$ and $y-$axes

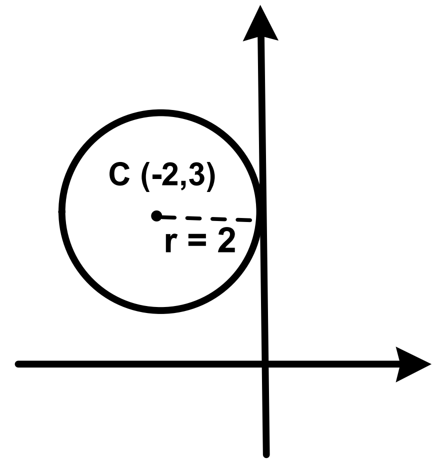
(b) It is tangent to the $x-$axis only

(c) It is neither tangent to the $x-$axix nor to the $y-$axis

✓(d) It is tangent to the $y-$ axis only

(e) It passes through the origin

Ans: $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 4 \implies (x+2)^2 + (y-3)^2 = 4$. The center is $C(-2,3)$ and radius r $= 2$. Clearly from the graph, the circle is tangent to the $y-$axis only.



17. One factor of the expression $x^2 - 10xy + 25y^2 - x + 5y$ is

(a) $x - 5y + 1$ (b) $x - 5y - 5$ ✓(c) $x - 5y - 1$ (d) $x + 5y + 1$ (e) $x + 5y - 5$

Ans: $= (x^2 - 10xy + 25y^2) + (-x + 5y) = (x - 5y)^2 - (x - 5y) = (x - 5y)(x - 5y - 1)$.

So $x - 5y - 1$ is one of the factors.

18. The Polynomial $P(x) = -2(x+1)(x-1)^2(3-x)$ has

(a) one relative minimum and one relative maximum

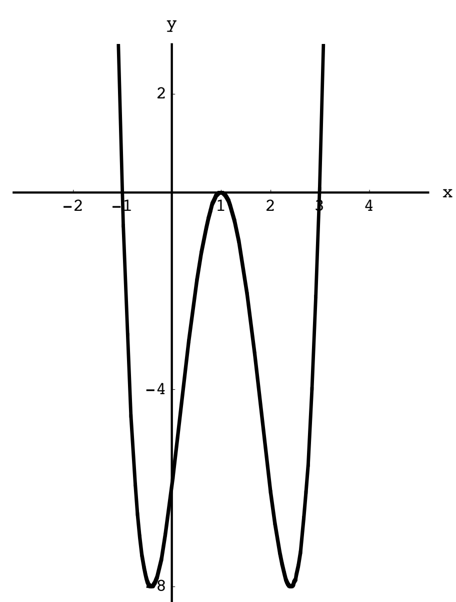
(b) two relative minima and no relative maximum

✓(c) two relative minima and one relative maximum

(d) no relative minimum and one relative maximum

(e) one relative minimum and two relative maxima

Ans: The zeros are -1 , 1 (of multiplicity two) and $3 \implies x-$intercepts are $(-1,0), (1,0)$ and $(3,0)$. If $x = 0$, then $y = -6 \implies y-$intercept $= (0,-6)$. The leading term $= -2(x)(x^2)(-x) = 2x^4$, so $y \to \infty$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$. Therefore the graph goes up to far-left and up to far-right. Moreover, the graph crosses the $x-$axis at $x = -1$, $x = 3$ (for odd powers) and touches the $x-$axis at $x = 1$ (for even power). Clearly from the graph, $P(x)$ has twor relative minima and one relative maximum.



19. If k is the solution of the equation $\sqrt{2x-3} - \sqrt{x+2} = 1$, then $2k+1 =$

(a) 5 (b) 31 (c) 15 ✓(d) 29 (e) 11

Ans: $\sqrt{2x-3} = 1 + \sqrt{x+2} \implies 2x-3 = 1 + 2\sqrt{x+2} + x + 2 \implies x - 6 = 2\sqrt{x+2} \implies x^2 - 12x + 36 = 4x + 8 \implies x^2 - 16x + 28 = 0 \implies (x-2)(x-14) = 0 \implies x = 2$ or $x = 14$. We must check both values. For $x = 2$: $\sqrt{1} - \sqrt{4} \stackrel{?}{=} 1$ (no) and for $x = 14$: $\sqrt{25} - \sqrt{16} \stackrel{?}{=} 1$ (yes) $\implies$ S.S. $= \{14\}$. So $k = 14 \implies 2k+1 = 28+1 = 29$

20. The equations of the vertical asymptote and the slant asymptote of the graph of $f(x) = \dfrac{x^3+1}{2x^2+x-1}$ respectively are

(a) $x = \dfrac{1}{2}$ and $y = \dfrac{1}{2}x - \dfrac{3}{4}$ (b) $x = 1$ and $y = \dfrac{1}{2}x - \dfrac{1}{4}$

(c) $x = -1$ and $y = \dfrac{1}{2}x + \dfrac{1}{4}$ ✓(d) $x = \dfrac{1}{2}$ and $y = \dfrac{1}{2}x - \dfrac{1}{4}$

(e) $x = \dfrac{1}{2}$ and $y = 2x - \dfrac{1}{4}$

Ans: $f(x) = \dfrac{\cancel{(x+1)}\,(x^2-x+1)}{\cancel{(x+1)}\,(2x-1)} = \dfrac{x^2-x+1}{2x-1}$. The vertial asymptote is $x = \dfrac{1}{2}$

and the slant asymptote is $y = \dfrac{1}{2}x - \dfrac{1}{4}$ (divide $x^2 - x + 1$ by $2x - 1$)

21. If we find the inverse function of $f(x) = \dfrac{3x+5}{7x+2}$, we get $f^{-1}(x) = \dfrac{2x+b}{cx+d}$, where $2d - bc =$

(a) -33 (b) 25 ✓(c) -29 (d) 31 e) -21

Ans: In the equation of f, interchange x with y to get $x = \dfrac{3y+5}{7y+2}$. Next solve for $y \implies 7xy + 2x = 3y + 5 \implies 7xy - 3y = 5 - 2x \implies (7x-3)y = 5 - 2x \implies y = f^{-1}(x) = \dfrac{5-2x}{7x-3} = \dfrac{2x-5}{-7x+3} \implies b = -5,\ c = -7,\ d = 3 \implies 2d - bc = 2(3) - (-5)(-7) = 6 - 35 = -29$

22. If one-half of a number minus one-fourth of the number is four more than one-fifth of the number, then the number is

✓(a) 80 (b) 90 (c) 100 (d) 70 (e) 60

Ans: Let the number be x. So $\dfrac{x}{2} - \dfrac{x}{4} = \dfrac{x}{5} + 4 \implies \dfrac{x}{4} = \dfrac{x}{5} + 4$ (multiply by 20)

$\implies 5x = 4x + 80 \implies x = 80$

23. The solution set of the equation $\dfrac{2|x+2|}{3} - \dfrac{1}{2} = \dfrac{4x+5}{6}$ is equal to

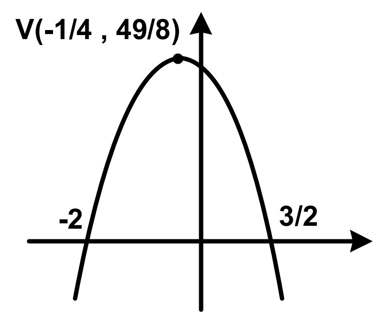
(a) $(-\infty, -2]$ (b) $\{-2\}$ ✓(c) $[-2, \infty)$ (d) $(-\infty, \infty)$ (e) ϕ

Ans: Multiply by 6 to get $4|x+2|-3 = 4x+5 \implies 4|x+2| = 4x+8 = 4(x+2) \implies |x+2| = x+2$ which is always true when $x+2 \geq 0 \implies x \geq -2 \implies$ S.S $= [-2, \infty)$

24. The maximum of the product $(3-2x)(x+2)$ is

(a) 4 (b) 6 (c) $\dfrac{49}{4}$ (d) $\dfrac{45}{4}$ ✓(e) $\dfrac{49}{8}$

Ans: Let $y = (3-2x)(x+2) = -2x^2 - x + 6 = -2(x^2 + \frac{1}{2}x + \frac{1}{16}) + 6 + \frac{1}{8} = -2(x + \frac{1}{4})^2 + \frac{49}{8}$. This is a parabola opens downward with vertex $V(-\frac{1}{4}, \frac{49}{8})$. Thus the maximum of the product $= \dfrac{49}{8}$



25. The $y-$intercept of the line passing through the points $(5, -6)$, and $(2, -1)$ is

(a) $\left(0, -\dfrac{11}{3}\right)$ (b) $\left(0, \dfrac{11}{3}\right)$ (c) $\left(0, -\dfrac{3}{7}\right)$ (d) $\left(0, \dfrac{8}{3}\right)$ ✓(e) $\left(0, \dfrac{7}{3}\right)$

Ans: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{-1+6}{2-5} = -\dfrac{5}{3}$. The equation is $-\dfrac{5}{3} = \dfrac{y+1}{x-2} \implies$

$$3y + 3 = -5x + 10 \implies 3y = -5x + 7 \implies y = -\frac{5}{3}x + \frac{7}{3} = mx + b \implies b = \frac{7}{3}$$

$$\implies y-\text{intercept} = (0, b) = \left(0, \frac{7}{3}\right)$$

26. If $z = (2-i)^2 + \sqrt{-4} \cdot \sqrt{-16}$, where $i = \sqrt{-1}$, then the conjugate $\bar{z}$ is equal to

(a) $5+4i$ (b) $-11+4i$ (c) $5-4i$ ✓(d) $-5+4i$ (e) $11+4i$

Ans: $z = (4 - 4i + i^2) + (2i)(4i) = 4 - 4i - 1 - 8 = -5 - 4i \implies \bar{z} = -5 + 4i$

27. If the distance between the points $(-x-2y, y-4x)$ and $(-x, 5y-4x)$, where $y > 0$, is $\sqrt{5}$, then $y =$

(a) $\frac{1}{5}$ (b) $\frac{\sqrt{2}}{2}$ ✓(c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) $\frac{3}{4}$

Ans: $\sqrt{(-x-2y+x)^2+(y-4x-5y+4x)^2} = \sqrt{5} \Longrightarrow \sqrt{(-2y)^2+(-4y)^2} = \sqrt{5}$

$\Longrightarrow \sqrt{4y^2+16y^2} = \sqrt{5} \Longrightarrow \sqrt{20y^2} = \sqrt{5} \Longrightarrow 2\sqrt{5}\,|y| = \sqrt{5} \Longrightarrow 2\sqrt{5}\,y = \sqrt{5}$

$\Longrightarrow 2y = 1 \Longrightarrow y = \frac{1}{2}$

28. If k is the number of possible negative real zeros of $P(x) = 2x^3 + x^2 - 5x + 1$ and l is the smallest positive integer that is an upper bound for the zeros of $P(x)$, then the $k + l =$

✓(a) 3 (b) 5 (c) 2 or 4 (c) 1 or 3 (e) 2

Ans: $P(-x) = -2x^3 + x^2 + 5x + 1$ has 1 variation in sign $\Longrightarrow P(x)$ has 1 negative real zero $\Longrightarrow k = 1$. When searching for l, we start with 1. Clearly from the adjacent figure 1 is not the correct value, while 2 is. This is because all the numbers in the bottom row are nonnegative real numbers $\Longrightarrow l = 2 \Longrightarrow k + l = 1 + 2 = 3$

1	2	1	-5	1
		2	3	-2
	2	3	-2	-1

2	2	1	-5	1
		4	10	10
	2	5	5	11

29. If $f(x) = \begin{cases} -\frac{1}{2}x & \text{if} \quad x < -2 \\ x^2 & \text{if} \quad -2 \le x < 1 \\ 4 - x & \text{if} \quad x \ge 1 \end{cases}$, then $(f \circ f)\left(-\frac{3}{2}\right) + (f \circ f)(-\frac{15}{2}) =$

(a) 0 (b) -2 (c) -1 (d) 1 ✓(e) 2

Ans: $= f\Big(f(-3/2)\Big) + f\Big(f(-15/2)\Big) = f\left(\frac{9}{4}\right) + f\left(\frac{15}{4}\right) = \left(4 - \frac{9}{4}\right) + \left(4 - \frac{15}{4}\right)$

$= \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$

30. The graph of the equation $y^2 = x^2 + 6x - 2y + 8$ can be obtained by shifting the graph of $y^2 = x^2$ (Hint: Complete the square)

(a) down vertically 3 units and left horizontally 1 unit

✔(b) down vertically 1 unit and left horizontally 3 units

(c) down vertically 1 unit and right horizontally 2 units

(d) up vertically 1 unit and right horizontally 3 units

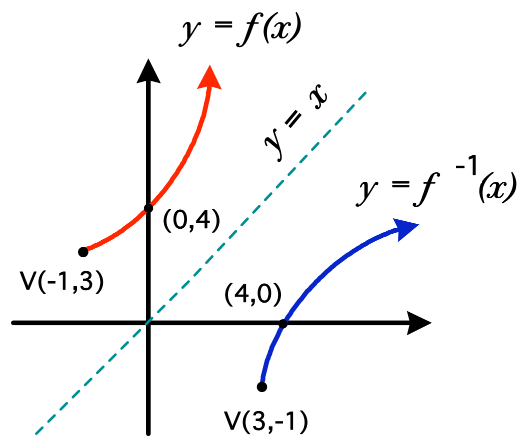
(e) down vertically 2 units and left horizontally 3 units

Ans: $y^2 + 2y = x^2 + 6x + 8 \Longrightarrow (y^2 + 2y + 1) = (x^2 + 6x + 9) + 8 + 1 - 9 \Longrightarrow (y+1)^2 = (x+3)^2$. Thus the graph of our equation can be obtained by shifting the graph of $y^2 = x^2$ down vertically 1 unit and left horizontally 3 units.

31. If $f(x) = x^2 + 2x + 4,\ x \geq -1,$ then the graph of f^{-1} lies completely above the $x-$axix on the interval

✔(a) $(4, \infty)$ (b) $(-1, 3)$ (c) $(3, \infty)$ (d) $(-\infty, 4)$ (e) $(3, 4)$

Ans: $y = f(x) = (x^2 + 2x + 1) + 4 - 1 = (x+1)^2 + 3,\ x \geq -1$. This is the right half of a parabola opens upward and with vertex $V(-1, 3)$. Remember that if $(a, b) \in$ the graph of f, then $(b, a) \in$ the graph of f^{-1}. To graph f^{-1}, just reflect the graph of f across the line $y = x$. Clearly from the adjacent figure, the graph of f^{-1} lies completely above the $x-$axix on the interval $(4, \infty)$.



32. The set of all real numbers k for which $x - 2$ is a factor of $f(x) = x^4 - kx^3 - 4x^2 + 8k$ contains

(a) the number zero only (b) no real numbers ✔(c) all real numbers

(d) all integers only (e) all rational numbers only

Ans: 2 is a zero of $f(x) \Longrightarrow$ the remainder of dividing $P(x)$ by $x - 2$ is r = 0. When synthetic division is used, we can see that the remainder is always 0 regardless of the value of k. This means that k can be any real number.

2	1	- k	- 4	0	8k
		2	4 - 2k	-4k	-8k
	1	2 - k	-2k	-4k	0

33. The graph of the function $f(x) = \dfrac{x^3 + x}{x^3 - x}$ is decreasing on the interval D and has an open circle (hole) at (a, b) where [Hint: sketch]

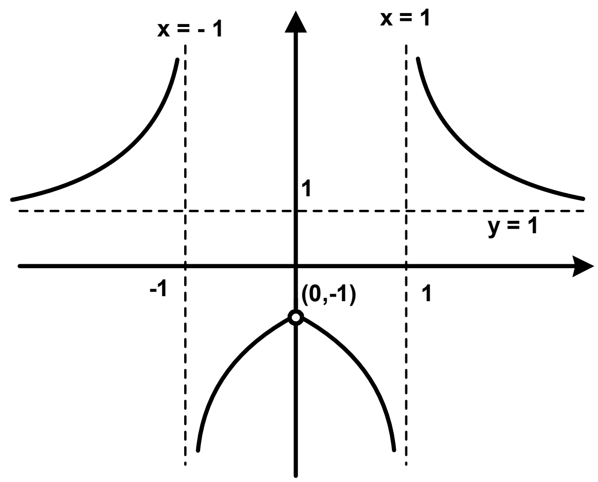
(a) $D = (-1,0) \cup (0,1),\ a = 0,\ b = -1$ ✓(b) $D = (0,1) \cup (1,\infty),\ a = 0,\ b = -1$

(c) $D = (0,1) \cup (1,\infty),\ a = 0,\ b = 2$ (d) $D = (-\infty,-1) \cup (1,\infty),\ a = 1,\ b = 0$

(e) $D = (-\infty,-1) \cup (-1,0),\ a = 0,\ b = -1$

Ans: $y = f(x) = \dfrac{\cancel{x}(x^2+1)}{\cancel{x}(x^2-1)} = \dfrac{x^2+1}{(x+1)(x-1)}$, $x \neq -1, x \neq 0$ and $x \neq 1$. The graph has a hole at $(0,-1) \Longrightarrow a = 0$ and $b = -1$. The graph has neither $x-$intercept (for $x^2 + 1 \neq 0$) nor $y-$intercept (for $x \neq 0$). The vertical asymptotes are $x = -1$ and $x = 1$. The horizontal asymptote is $y = 1$. From the below graph, f is decreasing on the interval $D = (0,1) \cup (1,\infty)$

(x^2+1)	+ +		+ +		+ +
$(x+1)$	− −		+ +		+ +
$(x-1)$	− −		− −		+ +
y is :	+	-1	−	1	+

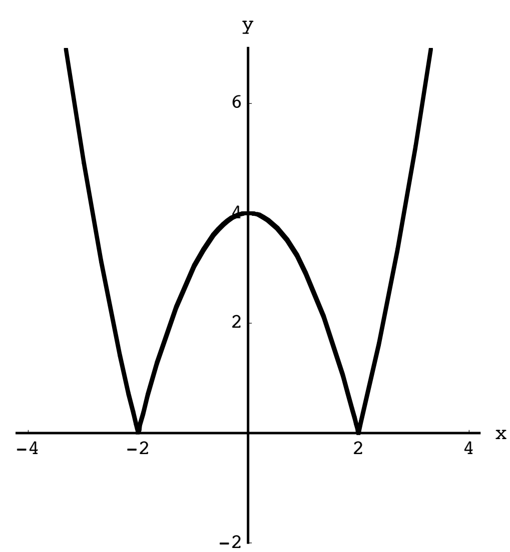


34. The function $f(x) = |x^2 - 4|$ can be made one-to-one over one of the intervals [Hint: sketch]

(a) $(-\infty, 0]$ (b) $(-\infty,-2] \cup [2,-\infty)$ ✓(c) $[2,\infty)$ (d) $[0,\infty)$ (e) $[-2,2]$

Ans: $y = \begin{cases} x^2 - 4 & \text{if } x^2 - 4 \geq 0 \\ 4 - x^2 & \text{if } x^2 - 4 < 0 \end{cases}$

$y = x^2 - 4$ when $x^2 \geq 4 \Longrightarrow |x| \geq 2 \Longrightarrow x \leq -2$ or $x \geq 2$. Its graph is the upper parts of a parabola opens upward and with vertex $V_1(0,-4)$. While $y = 4 - x^2$ when $x^2 < 4 \Longrightarrow |x| < 2 \Longrightarrow$ $-2 < x < 2$. Its graph is the upper part of a parabola opens downward and with vertex $V_2(0,4)$. Clearly from the graph, f is ono-to-one over the interval $[2,\infty)$ while it is not so over the other intervals.



35. If the discriminant of the quadratic equation $2x^2 + \frac{3}{5}x = k$ is $\frac{49}{25}$, where k is a constant, then the solution set of the equation contains

✔(a) one positive and one negative rational numbers

(b) two positive rational numbers

(c) two negative rational numbers

(d) two positive irrational numbers

(e) two negative irrational numbers

Ans: Discriminant $= b^2 - 4ac = \frac{49}{25}$. The equation has solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\frac{3}{5} \pm \sqrt{\frac{49}{25}}}{4} = \frac{-\frac{3}{5} \pm \frac{7}{5}}{4} = \frac{-10/5}{4} = -\frac{1}{2}$$

or $x = \frac{4/5}{4} = \frac{1}{5}$. So it has one positive and one negative rational numbers as solutions.