Math 001 **Solution of First Exam** **Semester I, Term 061**

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1. If $i=\sqrt{-1}$ and $z=1+i\sqrt{3}$, then $\frac{1}{i}(z^2-2z)$ is equal to

   ✓(a) $4i$ (b) $6i$ (c) $-2+3i$ (d) $-3i$ (e) $1-3i$

   <u>Ans:</u> $=\frac{1}{i}\left[(1+i\sqrt{3})^2-2(1+i\sqrt{3})\right]=\frac{1}{i}\left[(1+2\sqrt{3}i-3)-2-2\sqrt{3}i\right]=\frac{-4}{i}\cdot\frac{-i}{-i}=\frac{4i}{1}=4i$

2. One of the factors of $4x^2+4x+1-y^2$ is

   (a) $2x+y-1$ (b) $4x-y-1$ (c) $2x+y$ (d) $2x-y$ ✓(e) $2x-y+1$

   <u>Ans:</u> $(4x^2+4x+1)-y^2=(2x+1)^2-y^2=(2x+1-y)(2x+1+y)=(2x-y+1)(2x+y+1)$, so $2x-y+1$ is one of the factors.

3. The degree n and the leading coefficient L of the polynomial $(2-3x^2-x)^3(2x+5)$ are

   (a) $n=6$, $\mathrm{L}=-27$ ✓(b) $n=7$, $\mathrm{L}=-54$ (c) $n=7$, $\mathrm{L}=27$

   (d) $n=6$, $\mathrm{L}=54$ (e) $n=7$, $\mathrm{L}=-18$

   <u>Ans:</u> In $(2-3x^2-x)^3=(-3x^2-x+2)^3$, the leading term is $(-3x^2)^3=-27x^6$. So, in the product $(2-3x^2-x)^3(2x+5)$, the leading term is $(-27x^6)(2x)=-54x^7$.
   Therefore, the degree $=n=7$ and the leading coefficient $=\mathrm{L}=-54$

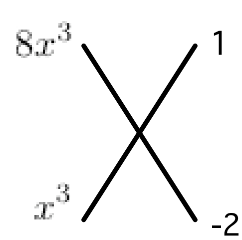
4. One of the factors of $8x^6-15x^3-2$ is

   ✓(a) $4x^2-2x+1$ (b) $2x^2-4x+2$ (c) $4x^3-2x+2$ (d) $2x^2-4x-1$ (e) $4x^2-2x-2$

   <u>Ans:</u> $=(8x^3+1)(x^3-2)$

   $=(2x+1)(4x^2-2x+1)(x^3-2)$

   So, $4x^2-2x+1$ is one of the factors.

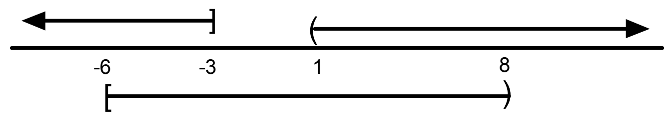


5. If $\mathrm{A} = \{x|x \le -3\} \cup \{x|x > 1\}$ and $\mathrm{B} = \{x| -6 \le x < 8\}$ then $\mathrm{A} \cap \mathrm{B} =$

(a) $\{x| -6 \le x \le -3\}$ (b) $\{x| -3 \le x < 1\} \cup \{x|1 < x < 8\}$

✓(c) $\{x| -6 \le x \le -3\} \cup \{x|1 < x < 8\}$ (d) $\{x| -6 \le x < 1\}$ (e) $\{x| -3 \le x < 8\}$

<u>Ans:</u> Clearly from the adjacent figure
$\mathrm{A} \cap \mathrm{B} = [-6, -3] \cup (1, 8)$
$= \{x| -6 \le x \le -3\} \cup \{x|1 < x < 8\}$



6. The **conjugate** of the complex number $\dfrac{8 + i^7}{2 + 3i^{13}}$ in standard form is

(a) $3 - 2i$ (b) $\frac{3}{13} - \frac{5}{13}i$ (c) $2 - i$ ✓(d) $1 + 2i$ (e) $\frac{3}{13} + \frac{5}{13}i$

<u>Ans:</u> $\mathrm{Z} = \dfrac{8 + i^3}{2 + 3i} = \dfrac{8 - i}{2 + 3i} \cdot \dfrac{2 - 3i}{2 - 3i} = \dfrac{(16 - 3) + (-24 - 2)i}{4 + 9}$

$= \dfrac{13 - 26i}{13} = 1 - 2i \implies \overline{\mathrm{Z}} = 1 + 2i$

7. $\dfrac{\dfrac{3y}{y-5} - \dfrac{2}{y-5}}{2(y-2)^{-1} + y^{-1}} =$

(a) $(y-2)(y-5)$ (b) $\dfrac{y}{(y-2)(y-5)}$ (c) $\dfrac{y(y+5)}{y-2}$ (d) $\dfrac{(y-2)(y-5)}{y}$ ✓(e) $\dfrac{y(y-2)}{y-5}$

<u>Ans:</u> $= \dfrac{\dfrac{3y-2}{y-5}}{\dfrac{2}{y-2} + \dfrac{1}{y}} \cdot \dfrac{y(y-2)(y-5)}{y(y-2)(y-5)} = \dfrac{y(3y-2)(y-2)}{2y(y-5) + (y-2)(y-5)} = \dfrac{y(3y-2)(y-2)}{(y-5)(2y + y - 2)}$

$= \dfrac{y\,\cancel{(3y-2)}\,(y-2)}{(y-5)\,\cancel{(3y-2)}} = \dfrac{y(y-2)}{y-5}$

8. $\dfrac{3x-4}{4x-1} - \dfrac{3x+6}{(1-4x)(x+2)} =$

(a) $\dfrac{3x-1}{x+2}$ (b) $\dfrac{3x-5}{(4x-1)(x+2)}$ (c) $\dfrac{3x-2}{(4x-1)^2(x+2)}$ ✓(d) $\dfrac{3x-1}{4x-1}$ (e) $\dfrac{3x-1}{(4x-1)^2(x+2)}$

<u>Ans:</u> $= \dfrac{3x-4}{4x-1} + \dfrac{3\,\cancel{(x+2)}}{(4x-1)\,\cancel{(x+2)}} = \dfrac{3x-4}{4x-1} + \dfrac{3}{4x-1} = \dfrac{3x-4+3}{4x-1} = \dfrac{3x-1}{4x-1}$

9. If $x \neq 0$, then the expression $\left[\dfrac{(3x^2)^{-1}(3x^5)^{-2}}{(3^{-1}x^{-2})^2}\right]^{-1}$ is equal to

(a) $27x^4$ (b) $9x^3$ ✓(c) $3x^8$ (d) $9x^6$ (e) $3x^4$

Ans: $= \dfrac{(3^{-1}x^{-2})^2}{(3x^2)^{-1}(3x^5)^{-2}} = (3x^2)(3x^5)^2(3^{-1}x^{-2})^2 = (3x^2)(9x^{10})\left(\dfrac{1}{9x^4}\right) = \dfrac{27x^{12}}{9x^4} = 3x^8$

10. $\dfrac{y^2+7y+12}{y^3-3y^2+9y} \div \dfrac{y^2+6y+9}{y^3+27} =$

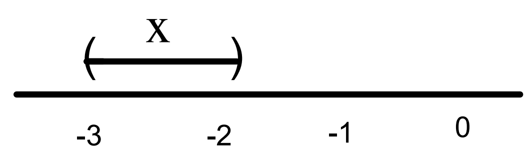
(a) $\dfrac{y+3}{y+4}$ (b) $\dfrac{y+3}{y-3}$ (c) $\dfrac{y+4}{y+3}$ ✓(d) $\dfrac{y+4}{y}$ (e) $\dfrac{y+3}{y}$

Ans: $= \dfrac{(y+3)\,(y+4)}{y\,(y^2-3y+9)} \cdot \dfrac{(y+3)(y^2-3y+9)}{(y+3)^2} = \dfrac{y+4}{y}$

11. Write without absolute values and simplify: $|-3x| + \sqrt{(x-3)^2} + 2|x+1|,\ -3 < x < -2$

(a) 1 (b) $4x+1$ (c) $5-2x$ ✓(d) $1-6x$ (e) $2x-5$

Ans: $= |3x| + |x-3| + 2|x+1|$



Now $3x < 0 \implies |3x| = -3x$, $x < 0 \implies x - 3 < 0 \implies |x-3| = -(x-3)$ and
$x < -2 \implies x < -1 \implies x+1 < 0 \implies |x+1| = -(x+1)$
Therefore the expression $= -(3x) - (x-3) - (x+1) = -3x - x + 3 - 2x - 2 = 1 - 6x$

12. If P and Q are any two different polynomials each of degree $n > 1$, then which one of the following statements is **Always True**?

(a) P − Q is a polynomial of degree $< n$

(b) P + Q is a polynomial of degree n

(c) P + P is a polynomial of degree $2n$

✓(d) P − Q is a polynomial of degree $\leq n$

(e) P Q is a polynomial of degree n^2

Ans: Degree of P − Q and P + Q $\leq n$. Degree of P + P $= n$. While degree of P Q $= n+n = 2n$.
So the correct answer is (d).

13. If $2^{x-1} = y$, then $2^{3x-2} =$

(a) $\frac{y^3}{8}$ (b) $\frac{y^3}{4}$ (c) $\frac{y^3}{2}$ (d) $4y^3$ ✔(e) $2y^3$

Ans: $2^{x-1} = y \implies 2^x \cdot 2^{-1} = y \implies \frac{2^x}{2} = y \implies \boxed{2^x = 2y}$

Thus $2^{3x-2} = 2^{3x} \cdot 2^{-2} = \frac{(2^x)^3}{4} = \frac{(2y)^3}{4} = \frac{8y^3}{4} = 2y^3$

14. Which one of the following statements is **Always True**?

✔(a) The product of two prime numbers is a composite number.
(b) The sum of two prime numbers is a prime number.
(c) Every rational number has a multiplicative inverse.
(d) The product of two irrational numbers is an irrational number.
(e) The sum of two irrational numbers is an irrational number.

Ans: (a) True, because the product of two prime numbers is a number divisible by both of them.
(b) False, becuse 3 , 5 are prime numbers, but $3 + 5 = 8$ is not a prime number.
(c) False, becuse 0 has no multiplicative inverse.
(d) False, becuse $\sqrt{3}$ is an irrational number, but $(\sqrt{3})(\sqrt{3}) = 3$ is rational.
(e) False, becuse $\sqrt{2}$, $-\sqrt{2}$ are irrational numbers, but $\sqrt{2} + (-\sqrt{2}) = 0$ is rational.

15. $\frac{-2}{1 + 2\sqrt{12} - 3\sqrt{3}} =$

(a) $1 - 2\sqrt{3}$ (b) $5 - \sqrt{3}$ ✔(c) $1 - \sqrt{3}$ (d) $2 + \sqrt{3}$ (e) $-2 - \frac{5}{9}\sqrt{3}$

Ans: $= \frac{-2}{1 + 4\sqrt{3} - 3\sqrt{3}} = \frac{-2}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{-2(1 - \sqrt{3})}{1 - 3} = \frac{-2(1 - \sqrt{3})}{-2} = 1 - \sqrt{3}$