Math 001 — Solution of Second Exam — Semester I, Term 061

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1. The graph of the equation $y^3 = -x^3y^2 + \dfrac{x}{|x|}$ is symmetric with respect to

(a) the x–axis and the origin (b) the x–axis only ✔(c) the origin only

(d) the y–axis and the origin (e) the y–axis only

<u>**Ans:**</u> Remember that $(-y)^3 = -y^3$, $(-x)^2 = x^2$, and $|-x| = |x|$

With respect to the x–axis: replace y by $(-y)$ to get $-y^3 = -x^3y^2 + \dfrac{x}{|x|} \Longrightarrow$
$y^3 = x^3y^2 - \dfrac{x}{|x|}$. There is a change (not symmetric).

With respect to the y–axis: replace x by $(-x)$ to get $y^3 = -(-x^3)(y^2) + \dfrac{-x}{|-x|} \Longrightarrow$
$y^3 = x^3y^2 - \dfrac{x}{|x|}$. There is a change (not symmetric).

With respect to the origin: replace x by $(-x)$ and y by $(-y)$ to get
$-y^3 = -(-x^3)(y^2) + \dfrac{-x}{|-x|} \Longrightarrow y^3 = -x^3y^2 + \dfrac{x}{|x|}$. There is no change (symmetric).

2. If $x < 0$, then the distance between the points $P(2x, -7x)$ and $Q(-2x, -4x)$ is equal to

(a) $5x$ (b) $7x$ (c) $-3x$ (d) $-7x$ ✔(e) $-5x$

<u>**Ans:**</u> $d(P,Q) = \sqrt{(2x+2x)^2 + (-7x+4x)^2} = \sqrt{(4x)^2 + (-3x)^2} = \sqrt{16x^2 + 9x^2} = \sqrt{25x^2}$

$= |5x| = 5|x| = -5x$, because $x < 0$

3. The center $C(h,k)$ and the radius r of the circle $\dfrac{1}{2}x^2 + \dfrac{1}{2}y^2 - 3x + 2y - \dfrac{3}{2} = 0$ are

(a) $C(3,-2)$, $r = \sqrt{17}$ ✔(b) $C(3,-2)$, $r = 4$ (c) $C(\frac{3}{2}, -1)$, $r = \dfrac{\sqrt{2}}{2}$

(d) $C(3,-2)$, $r = \sqrt{15}$ (e) $C(2,-3)$, $r = 4$

<u>**Ans:**</u> Multiply the equation by 2 to get $x^2 - 6x + y^2 + 4y = 3 \Longrightarrow$

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 9 + 4 + 3 \Longrightarrow (x-3)^2 + (y+2)^2 = 16 \Longrightarrow C(3,-2)$, $r = 4$

4. The x–intercept of the line passing through the points $(5,-6)$ and $(2,-8)$ is

(a) $(18,0)$ ✓(b) $(14,0)$ (c) $(\frac{2}{3},0)$ (d) $(-\frac{28}{3},0)$ (e) 10

Ans: The slope $= \text{m} = \frac{\Delta y}{\Delta x} = \frac{-8+6}{2-5} = \frac{-2}{-3} = \frac{2}{3}$.

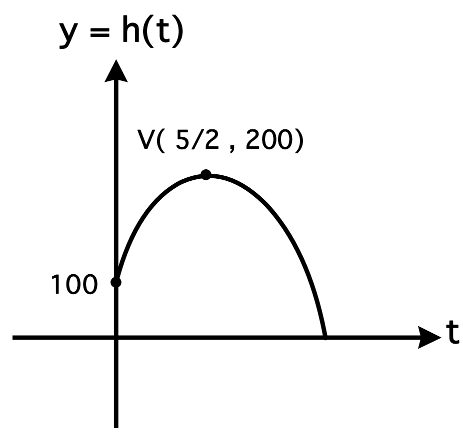
The equation of the line is $\frac{2}{3} = \frac{y+8}{x-2} \implies 2x-4 = 3y+24 \implies 2x-3y = 28$.

Put $y=0$, so $2x = 28 \implies x = 14 \implies$ x–intercept $= (14,0)$.

5. A ball is thrown vertically upward. If the height h (in feet) of the ball is given by the equation $h(t) = -16t^2 + 80t + 100$, where t is time in seconds, then the maximum height that the ball attains is

✓(a) 200 feet (b) 150 feet (c) 300 feet (d) 250 feet (e) 100 feet

Ans: $y = -16(t^2 - 5t + \frac{25}{4}) + 100 + 100$
$= -16(t - \frac{5}{2})^2 + 200$. This is a parabola opens upward with vertex $V(\frac{5}{2}, 200)$ and range $= [0,200]$,
Thus the maximum height $=$ 200 feet. (see the graph)



6. Identify the set of ordered pairs (x,y) or the relation which defines y as a function of x

(a) $y^2 = x^2$ (b) $|y| = x+5$ (c) $\left\{\left(\frac{1}{2},0\right), (2,-1), (3,3), \left(\frac{1}{2},\frac{1}{4}\right)\right\}$

✓(d) $5y + x = 2y + \sqrt{x^2-5}$ (e) $(x-1)^2 + (y-2)^2 = 25$

Ans: A relation is called a function if for each value of x in the domain, there is one value for y in the range. In part (a), when $x=1$, $y=\pm 1$. In part (b), when $x=0$, $y=\pm 5$. In part (c), when $x = \frac{1}{2}$, $y = 0$ or $\frac{1}{4}$. And in part (e), when $x=1$, $y=-3$ or 7. While in part (d), for any x in the domain, there is a single value for y.

7. If the lines $kx+4y=24$ and $y=-\frac{3}{k+1}\ x+\frac{15}{4}$ are parallel, then the set of values of k consists of

(a) two positive integers ✔(b) one positive and one negative integers

(c) one positive integer only (d) two negative integers (e) one negative integer only

Ans: $m_1=-\frac{k}{4}$ and $m_2=-\frac{3}{k+1}$. The two lines are parallel, so $m_1=m_2 \Longrightarrow -\frac{k}{4}=-\frac{3}{k+1} \Longrightarrow k^2+k=12 \Longrightarrow k^2+k-12=0 \Longrightarrow (k+4)(k-3)=0 \Longrightarrow k=-4$ or $k=3$

8. If the graph of the function $g(x)$ is obtained from the graph of $f(x)=\sqrt{x}$ by means of reflection across the x–axis , a horizental shift 2 units left and a vertical shift 1 unit upward, then $g(x)=$

(a) $\sqrt{x+2}-1$ (b) $-\sqrt{x+2}-2$ (c) $-\sqrt{x-2}+1$ ✔(d) $-\sqrt{x+2}+1$ (e) $\sqrt{1-x}+2$

Ans: Reflection across the x–axis produces the equation $y=-\sqrt{x}$, and with the horizental and vertical shifting, the obtained function is $(y-1)=-\sqrt{x+2} \Longrightarrow y=g(x)=-\sqrt{x+2}+1$

9. Which one of the following numbers is in the range of the quadratic function $f(x)=-2x^2+x-\frac{3}{8}$?

(a) 2 (b) $-\frac{1}{8}$ (c) $-\frac{1}{16}$ (d) 0 ✔(e) $-\frac{1}{3}$

Ans: $y=-2(x^2-\frac{1}{2}x+\frac{1}{16})+\frac{1}{8}-\frac{3}{8}=-2(x-\frac{1}{4})^2-\frac{1}{4}$. This is a parabola opens downward with vertex $V(\frac{1}{4},-\frac{1}{4})$. The range $=(-\infty,-\frac{1}{4}]$, cleary only $-\frac{1}{3}\in(-\infty,-\frac{1}{4}]$.

10. The equation of the line passes through the point $(3,5)$ and perpendicular to the line $2x+5y-4=0$ is

(a) $5x-2y-30=0$ ✔(b) $5x-2y-5=0$ (c) $5x-2y-25=0$

(d) $2x-5y+19=0$ (e) $5x-2y+15=0$

Ans: Slope of the first line $(L_1)=m_1=-\frac{2}{5}$. $L_1\perp L_2 \Longrightarrow m_2=-\frac{1}{m_2}=\frac{5}{2}$.

The equation of L_2 is $\frac{5}{2}=\frac{y-5}{x-3} \Longrightarrow 5x-15=2y-10 \Longrightarrow 5x-2y-5=0$

11. If $M(h, 6)$ is the midpoint of the line segment from the point $P(3, k)$ to the point $Q(-5, 4)$, then $h + k =$

(a) 9 ✓(b) 7 (c) 5 (d) -1 (e) 1

Ans: $M(h, 6) = M\left(\frac{-5+3}{2}, \frac{k+4}{2}\right) = M\left(-1, \frac{k+4}{2}\right) \implies h = -1$ and $\frac{k+4}{2} = 6$

$\implies k + 4 = 12 \implies k = 8 \implies h + k = -1 + 8 = 7$

12. The domain, in interval notation, of the function $f(x) = \sqrt{2 - x - x^2}$ is

(a) $(-\infty, -2) \cup [1, \infty)$ (b) $[-2, \infty)$ (c) $(-\infty, 1]$ (d) $(-\infty, -1] \cup [2, \infty)$ ✓(e) $[-2, 1]$

Ans: $2 - x - x^2 \geq 0 \implies x^2 + x - 2 \leq 0 \implies$

$(x + 2)(x - 1) \leq 0 \implies$ (by sign test)

Domain $= [-2, 1]$

(x +2)	- -		+ +		+ +
(x - 1)	- -		- -		+ +
L.S. is :	+	-2	-	1	+

13. If $[x]$ denotes the greatest integer less than or equal to x, then the domain D and range R of the function $f(x) = \big|[x]\big| + 1$ are given by

(a) $D = (-\infty, \infty)$, R = all nonnegative integers (b) $D = R = [1, \infty)$ (c) $D = R = (-\infty, \infty)$

✓(d) $D = (-\infty, \infty)$, R = all natural numbers (e) $D = [0, \infty)$, $R = [1, \infty)$

Ans: For the function $f(x) = \big|[x]\big| + 1$, x can be any real number, so $D = (-\infty, \infty)$.

The range of $y = [x]$ is the set of integers $= \{\ldots\ldots, -2, -1, 0, 1, 2, \ldots\ldots\}$

The range of $y = \big|[x]\big|$ is the set of whole numers $= \{0, 1, 2, 3, \ldots\ldots\}$

The range R of $y = \big|[x]\big| + 1$ is the set of natural numers $= \{1, 2, 3, \ldots\ldots\}$

14. Let f be a function such that $f(-1) = 3$ and $f(2) = -4$. The coordinates of two ponts on the graph of $y = 3f(-x) - 2$ are

(a) $(1, 7)$, $(2, 4)$ (b) $(1, 7)$, $(2, 2)$ (c) $(-1, 1)$, $(2, 6)$

(d) $(1, 1)$, $(-2, -14)$ ✓(e) $(1, 7)$, $(-2, -14)$

Ans:

Point on $y = f(x)$	Point on $y = 3f(-x) - 2$
(a, b)	$(-a, 3b - 2)$
$(-1, 3)$	$(1, 3 \star 3 \ - 2) = (1, 7)$
$(2, -4)$	$\left(-2, 3 \star (-4) \ - 2\right) = (-2, -14)$

15. The function $f(x) = \begin{cases} -2x + 1 & \text{if } \ x < -2 \\ -x^2 & \text{if } \ -2 \leq x \leq 2 \\ -4 & \text{if } \ x > 2 \end{cases}$

is increasing on the interval(s) [Hint: sketch]

(a) $[-2, \infty)$ (b) $(-\infty, -2] \cup [2, \infty)$ (c) $[-2, 0] \cup [2, \infty)$ ✓(d) $[-2, 0]$ (e) $(-\infty, -2]$

Ans: The graph of $y = -2x + 1$ is a slant (half) line L_1 with a right boundary point $(-2, 5) \notin L_1$ and the point $(-3, 7) \in L_1$. The graph of $y = -x^2$ is a parabola opens downward with vertex V$(0, 0)$ and endpoints $(-2, -4)$, $(2, -4)$. While the graph of $y = -4$ is a horizental line. Notice that the point $(2, -4)$ does not belong to the horizental line, but it belongs to the parabola and so it belongs to the graph of $f(x)$.