Math 001.12 **Quiz # 3** **Sem. I , Dec 05 , 2005**

Name : ______________ **ID. # :** ______ **SER. # :** ____

1. Find the value of k if the line $(k+2)x - 3y - 1 = 0$ is perpendicular to the line $6x - 3ky = 5$

(2 pts)

<u>**Ans**</u> : $\mathrm{m}_1 = \dfrac{-(k+2)}{-3} = \dfrac{k+2}{3}$, $\mathrm{m}_2 = \dfrac{-6}{-3k} = \dfrac{2}{k}$. Since $\mathrm{L}_1 \perp \mathrm{L}_2$, then $\mathrm{m}_1 \cdot \mathrm{m}_2 = -1$
$\Longrightarrow \ (\frac{k+2}{3})(\frac{2}{k}) = -1 \ \Longrightarrow \ 2k + 4 = -3k \ \Longrightarrow \ 5k = -4 \ \Longrightarrow \ k = -\frac{4}{5}$

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2. Find the **domain** of $f(x) = \dfrac{x}{\sqrt{x^2 - 4}}$ and the **range** of $y = g(x) = -2x^2 + 3x + 1$ (4 pts)

<u>**Ans**</u> : $\mathrm{D}_f : x^2 - 4 > 0 \ \Longrightarrow \ x^2 > 4 \ \Longrightarrow \ |x| > 2 \ \Longrightarrow \ x < -2$ or $x > 2 \ \Longrightarrow$
$\mathrm{D}_f = (-\infty, -2) \cup (2, \infty)$

$\mathrm{R}_g : \ y = -2(x^2 - \frac{3}{2}x + \frac{9}{16}) + 1 + \frac{9}{8} = -2(x - \frac{3}{4})^2 + \frac{17}{8}$.

This is a parabola opens downward with vertex $\mathrm{V}(\frac{3}{4}, \frac{17}{8})$. Therefore $\mathrm{R}_g = (-\infty, \frac{17}{8}]$

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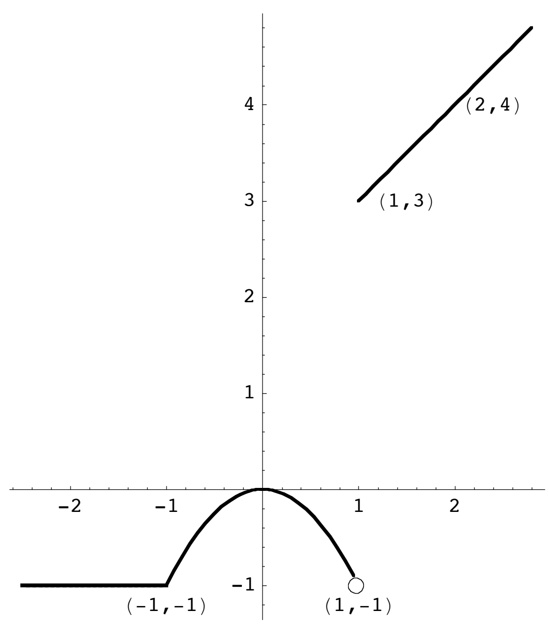
3. Graph $y = f(x) = \begin{cases} x + 2 & \text{if } \quad x \geq 1 \\ -x^2 & \text{if } \ -1 < x < 1 \\ -1 & \text{if } \quad x \leq -1 \end{cases}$, then find the (4 pts)

intervals on which $f(x)$ is increasing.

<u>**Ans**</u>: $y = x + 2$ is a slant line that contains the points $(1, 3)$, $(2, 4)$. $y = -x^2$ is a parabola with endpoints $(-1, -1)$, $(1, -1)$ (the second point is a boundary point). $y = -1$ is a horizental line. From the graph of $f(x)$, f is increasing on $(-1, 0) \cup (1, \infty)$



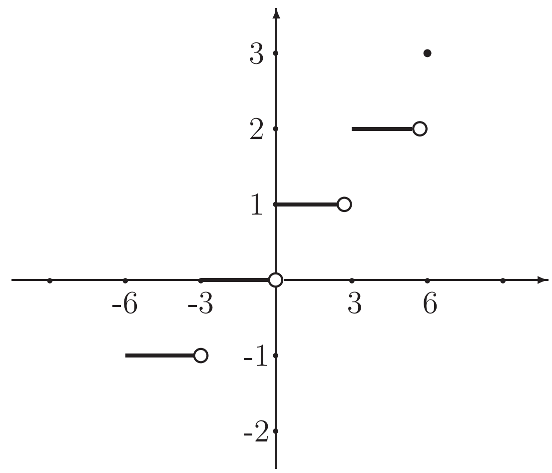
Math 001.18 **Quiz # 3** **Sem. I , Dec 05 , 2005**

Name :________________________ **ID. # :** ______ **SER. # :** ____

1. Graph $y = f(x) = [[\ \frac{x}{3}\ ]] + 1,\ -6 \leq x \leq 6$. Write the range of $f(x)$

   Ans: $-6 \leq x \leq 6 \implies -2 \leq \frac{x}{3} \leq 2$ (3 pts)

$y = [[\frac{x}{3}]] + 1$		
$\frac{x}{3}$	x	$y = [[\frac{x}{3}]] + 1$
$-2 \leq \frac{x}{3} < -1$	$-6 \leq x < -3$	$-2 + 1 = -1$
$-1 \leq \frac{x}{3} < 0$	$-3 \leq x < 0$	$-1 + 1 = 0$
$0 \leq \frac{x}{3} < 1$	$0 \leq x < 3$	$0 + 1 = 1$
$1 \leq \frac{x}{3} < 2$	$3 \leq x < 6$	$1 + 1 = 2$
$\frac{x}{3} = 2$	$x = 6$	$2 + 1 = 3$
Note: This graph can be obtained by stretching the graph of y=[[x]] horizontally by a factor of 3, then translating 1 unit upward		



Range $= \{-1, 0, 1, 2, 3\}$

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2. Find the **maximum** value and the **interval** on which the function
   $y = f(x) = -\frac{1}{2}x^2 + 5x + 1$ is **decreasing**. (3 pts)

   Ans: $y = -\frac{1}{2}(x^2 - 10x + 25) + 1 + \frac{25}{2} = -\frac{1}{2}(x-5)^2 + \frac{27}{2}$. This is a parabola opens downward with vertex $V(5, \frac{27}{2})$. So maximum value $= \frac{27}{2}$ and $f(x)$ is decreasing on the interval $(5, \infty)$.

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3. (a) Find the equation of the line through the point $(-1, 0)$ and perpendicular to the line $2x+3y = 5$

   (2 pts)

   Ans: $m_2 = -\frac{2}{3}$, since $L_1 \perp L_2$, then $m_1 = -\frac{1}{m_2} = \frac{3}{2}$.

   The equation is $\frac{3}{2} = \frac{y - 0}{x + 1} \implies 2y = 3x + 3 \implies 2y - 3x = 3$.

   (b) Let $f(x)$ be a linear function. If $f(1) = 4$ and $f(-2) = 1$, then find the x- and y-intercept of the graph of $f(x)$. (Hint: find $f(x)$ and no need for graph) (2 pts)

   Ans: $y = f(x) = ax + b$. $f(1) = 4 = a + b$ and $f(-2) = 1 = -2a + b$. Solving the two equations, we can get $3 = 3a \implies a = 1, b = 4 - a = 3 \implies \boxed{y = f(x) = x + 3}$. Now
   $y = 0 \implies x = -3 \implies$ x–intercept $= (-3, 0)$ and
   $x = 0 \implies y = 3 \implies$ y–intercept $= (0, 3)$

Math 001.30 **Quiz # 3** **Sem. I , Dec 05 , 2005**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Find the equation of the line through the point $(1,-2)$ and perpendicular to the line $\frac{3}{2}x = 5 - \frac{1}{2}y$.

(2 pts)

<u>**Ans**</u>: $\text{L}_2 : 3x = 10 - y \implies m_2 = -3$, since $\text{L}_1 \perp \text{L}_2$, then $m_1 = -\frac{1}{m_2} = \frac{1}{3}$.
The equation of L_1 is $\frac{1}{3} = \frac{y+2}{x-1} \implies x - 1 = 3y + 6 \implies x - 3y = 7$

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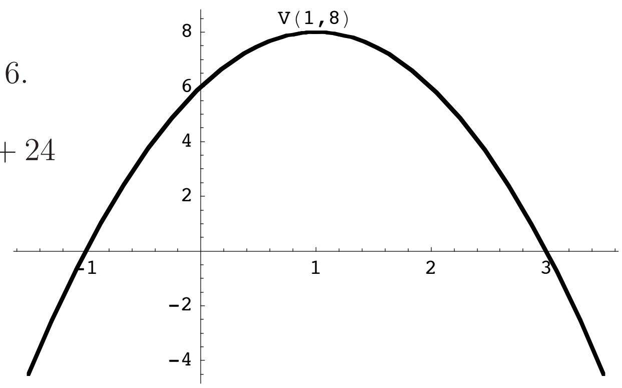
2. Let $f(x)$ be a quadratic function. If the graph of $f(x)$ has y-intercept at $(0,6)$ and x-intercepts at $(-1,0)$ and $(3,0)$, then find the range of $f(x)$. (Hint: find $f(x)$ and graph it) (4 pts)

<u>**Ans**</u>: $y = f(x) = ax^2 + bx + c$. The point $(0,6)$ lies on f
$\implies 6 = 0 + 0 + 6 \implies \boxed{c = 6}$, so $y = f(x) = ax^2 + bx + 6$.
The points $(-1,0)$ and $(3,0)$ lie on f, so $0 = a - b + 6$ and
$0 = 9a + 3b + 6$. Solving the two equations, we get $0 = 12a + 24$
$\implies \boxed{a = -2}$, $b = a + 6 \implies \boxed{b = 4}$.
Thus $y = f(x) = -2x^2 + 4x + 6 = -2(x^2 - 2x + 1) + 2 + 6$
$= -2(x-1)^2 + 8$. This is a parabola opens downward with
vertex $\text{V}(1,8) \implies$ the range $= (-\infty, 8]$



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3. (a) Find the domain of $y = f(x) = \frac{1}{\sqrt{|x+5| - 1}}$ (2 pts)

<u>**Ans**</u>: $|x+5| - 1 > 0 \implies |x+5| > 1 \implies x + 5 < -1$ or $x + 5 > 1$
$\implies x < -6$ or $x > -4 \implies \text{D} = (-\infty, -6) \cup (-4, \infty)$

(b) Solve the equation: $3[[\ 2x - 5\ ]] + 7 = 1$ (2 pts)

<u>**Ans**</u>: $\implies 3[[2x-5]] = -6 \implies [[2x-5]] = -2 \implies -2 \le 2x - 5 < -1$
$\implies 3 \le 2x < 4 \implies \frac{3}{2} \le x < 2 \implies \text{S.S.} = [\frac{3}{2}, 2)$