Math 001.12 **Quiz # 2** **Sem. I , Oct 25 , 2005**

Name : ____________________ **ID. # :** ______ **SER. # :** ____

1. For the quadratic equation $2x^2 + 3x + 6 = 0$: (4 pts)

   (i) Find the **sum and product** of the roots of the equation (without finding the roots).

   (ii) Solve the equation by **completing the square**.

   <u>**Ans**</u> : (i) Sum of roots $= -\frac{b}{a} = -\frac{3}{2}$, Product of roots $= \frac{c}{a} = \frac{6}{2} = 3$

   (ii) Divide by 3 to get: $x^2 + \frac{3}{2}x = -3$, then add $(\frac{1}{2} \cdot \frac{3}{2})^2 = \frac{9}{16}$ to both sides to get:

   $x^2 + \frac{3}{2}x + \frac{9}{16} = -3 + \frac{9}{16} = -\frac{39}{16} \implies (x + \frac{3}{4})^2 = -\frac{39}{16} \implies x + \frac{3}{4} = \pm\frac{\sqrt{39}}{4}\, i \implies \boxed{x = -\frac{3}{4} \pm \frac{\sqrt{39}}{4}\, i}$

   Notice that the sum of roots $= (-\frac{3}{4} + \frac{\sqrt{39}}{4}\, i) + (-\frac{3}{4} - \frac{\sqrt{39}}{4}\, i) = -\frac{6}{4} = -\frac{3}{2}$ and the product of roots $= (-\frac{3}{4} + \frac{\sqrt{39}}{4}\, i)(-\frac{3}{4} - \frac{\sqrt{39}}{4}\, i) = (-\frac{3}{4})^2 + (\frac{\sqrt{39}}{4})^2 = \frac{9}{16} + \frac{39}{16} = \frac{48}{16} = 3$ (see part (i))

---

2. Solve the equation $\sqrt{3x-5} - \sqrt{x+2} - 1 = 0$ (3 pts)

   <u>**Ans**</u> : $\sqrt{3x-5} = 1 + \sqrt{x+2} \xrightarrow[\text{both sides}]{\text{square}} 3x - 5 = 1 + 2\sqrt{x+2} + x + 2$

   $\implies 2x - 8 = 2\sqrt{x+2} \implies x - 4 = \sqrt{x+2} \xrightarrow[\text{again}]{\text{square}} x^2 - 8x + 16 = x + 2 \implies$

   $x^2 - 9x + 14 = 0 \implies (x-7)(x-2) = 0 \implies x = 7$ or $x = 2$. Now we must check both values:

   $x = 7$: $\sqrt{16} - \sqrt{9} - 1 \stackrel{?}{=} 0$ (yes)

   $x = 2$: $\sqrt{1} - \sqrt{4} - 1 \stackrel{?}{=} 0$ (no). Therefore S.S. $= \{7\}$

---

3. The denominator of a fraction is 4 more than the numerator. If the numerator is increased by 5 and the denominator is decreased by 7, the resulting number is 2, find the original fraction. (3 pts)

   <u>**Ans**</u> : Let the numerator $= x$, so the denominator $= x + 4$ and hence the fraction $= \frac{x}{x+4}$.

   Now $2 = \frac{x+5}{x+4-7} = \frac{x+5}{x-3} \implies 2x - 6 = x + 5 \implies x = 11 \implies$ the original fraction $= \frac{11}{15}$

Instructor: A. Al-Shallali

Math 001.18 **Quiz # 2** **Sem. I , Oct 25 , 2005**

Name : ______________________ **ID. # :** ______ **SER. # :** ____

1. Find the values of k for which the quadratic equation $2x^2 + kx + x + 1 = 0$ has two **equal real** roots. (3.5 pts)

   Ans : The quadratic equation $2x^2 + (k+1)x + 1 = 0$ has two equal real roots when the descriminant $= b^2 - 4ac = 0 \implies (k+1)^2 - 4(2)(1) = 0 \implies (k+1)^2 = 8 \implies k + 1 = \pm\sqrt{8} = \pm 2\sqrt{2}$

   $\implies \boxed{k = -1 \pm 2\sqrt{2}}$

---

2. Solve the equation $\sqrt{2x+4} + \sqrt{x+3} - 1 = 0$ (3.5 pts)

   Ans : $\sqrt{2x+4} = 1 - \sqrt{x+3} \xrightarrow[\text{both sides}]{\text{square}} 2x + 4 = 1 - 2\sqrt{x+3} + x + 3 \implies x = -2\sqrt{x+3} \xrightarrow[\text{again}]{\text{square}}$ $x^2 = 4x + 12 \implies x^2 - 4x - 12 = 0 \implies (x-6)(x+2) = 0 \implies x = 6$ or $x = -2$. Now we must check both values:

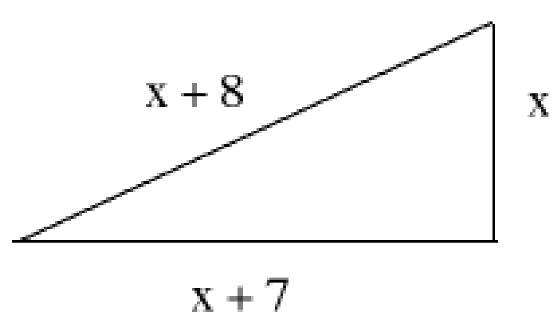
   $x = 6 :\ \sqrt{16} + \sqrt{9} - 1 \stackrel{?}{=} 0$ (no)

   $x = -2 :\ \sqrt{0} + \sqrt{1} - 1 \stackrel{?}{=} 0$ (yes). Therefore S.S. $= \{-2\}$

---

3. The sides of a **right triangle** are of length x , $x + 7$, and $x + 8$ cm. Find the perimeter and the area of the triangle. (3 pts)

   Ans : Notice that the hypotenuse is the largest side of any right triangle and so it has the length $x+8$ cm. By Pythagorean Theorem: $(x+8)^2 = x^2 + (x+7)^2 \implies x^2 + 16x + 64 = x^2 + x^2 + 14x + 49 \implies$ $x^2 - 2x - 15 = 0 \implies (x-5)(x+3) = 0 \implies x = 5$ or $x = -3$ (rejected). The sides are of length 5 , 12 , and 13 cm. The perimeter $= 5 + 12 + 13 = 30$ cm and the area $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(12)(5) = 30\,\text{cm}^2$.



Instructor: A. Al-Shallali

Math 001.30 **Quiz # 2** **Sem. I , Oct 25 , 2005**

Name : ____________________ **ID. # :** ______ **SER. # :** ____

1. Which one of the following equations is a **contradiction**? (Explain your answer) (3 pts)

   (i) $(x+3)(x-2)=1$ (ii) $(x-3)(x+3)+10=x^2+1$ (iii) $\dfrac{x+1}{x-1}=\dfrac{2}{x-1}$

   <u>Ans</u> : (iii) $\dfrac{x+1}{x-1}=\dfrac{2}{x-1},\ x\neq 1 \implies x+1=2 \implies x=1$ but $x\neq 1 \implies$ S.S. $=\phi$.
   Therefore this one is a contradiction while (i) is a conditional equation and (ii) is an identity, because L.S. $\equiv$ R.S. $=x^2+1$

---

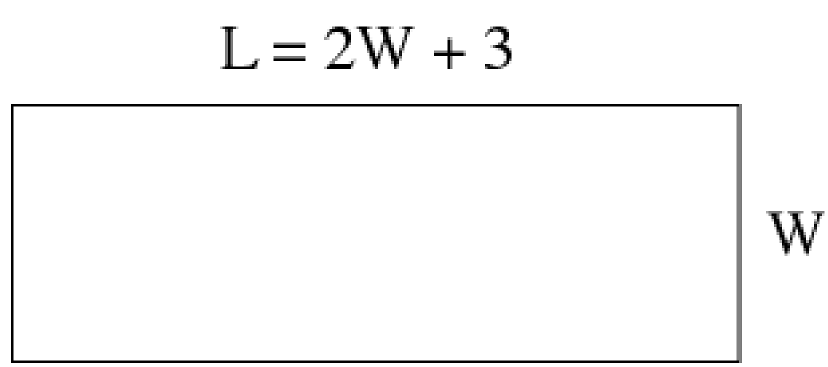
2. Solve the equation $\sqrt[3]{\sqrt{x+1}+\sqrt{2x+3}+7}=2$ (4 pts)

   <u>Ans</u> : $\xrightarrow[\text{both sides}]{\text{cube}} \sqrt{x+1}+\sqrt{2x+3}+7=8 \implies \sqrt{2x+3}=1-\sqrt{x+1} \xrightarrow[\text{both sides}]{\text{square}}$
   $2x+3=1-2\sqrt{x+1}+x+1 \implies x+1=-2\sqrt{x+1} \xrightarrow[\text{again}]{\text{square}} x^2+2x+1=4x+4 \implies$
   $x^2-2x-3=0 \implies (x-3)(x+1)=0 \implies x=3$ or $x=-1$. Now we must check both values:
   $x=3:\ \sqrt[3]{2+3+7}\overset{?}{=}2$ (no)
   $x=-1:\ \sqrt[3]{0+1+7}\overset{?}{=}2$ (yes). Therefore S.S. $=\{-1\}$

---

3. The length of a rectangle is 3 meters more than twice the width of the rectangle. If the area of the rectangle is 27, find the perimeter of the rectangle. (3 pts)

   <u>Ans</u> : Let the length = L and the width = W, then $\boxed{L=2W+3}$
   Area $=$ LW $=(2W+3)(W)=27 \implies 2W^2+3W-27=0 \implies (W-3)(2W+9)=0 \implies$
   $W=3$ or $W=-\frac{9}{2}$ (rejected).
   Therefore W = 3 cm , L = 9 cm and perimeter $=2(L+W)=2(9+3)=24$ cm.



Instructor: A. Al-Shallali