**Math 001.12** **Quiz # 4** **Sem. I , Jan 03 , 2006**

**Name :** ______________________ **ID. # :** ______ **SER. # :** ____

1. Let $p(x)$ be a polynomial with **real** coefficients. If $1+3i$ is a zero of $p(x) = x^4 - 2x^3 + 7x^2 + 6x - 30$, then find the **other zeros**. (5 pts)

<u>**Ans**</u>:

| | | | | | |
|---|---|---|---|---|---|
| $1+3i$ | 1 | $-2$ | 7 | 6 | $-30$ |
| | | $1+3i$ | $-10$ | $-3-9i$ | 30 |
| $1-3i$ | 1 | $-1+3i$ | $-3$ | $3-9i$ | 0 |
| | | $1-3i$ | 0 | $-3+9i$ | |
| | 1 | 0 | $-3$ | 0 | |

Since $p(x)$ has only real coefficients, then
$\overline{1+3i} = 1-3i$ is also a zero of $p(x)$ .

Let $q(x) = x^2 - 3 = 0 \implies x^2 = 3 \implies x = \pm\sqrt{3}$ .
The other zeros are $1-3i$ , $\sqrt{3}$ , and $-\sqrt{3}$

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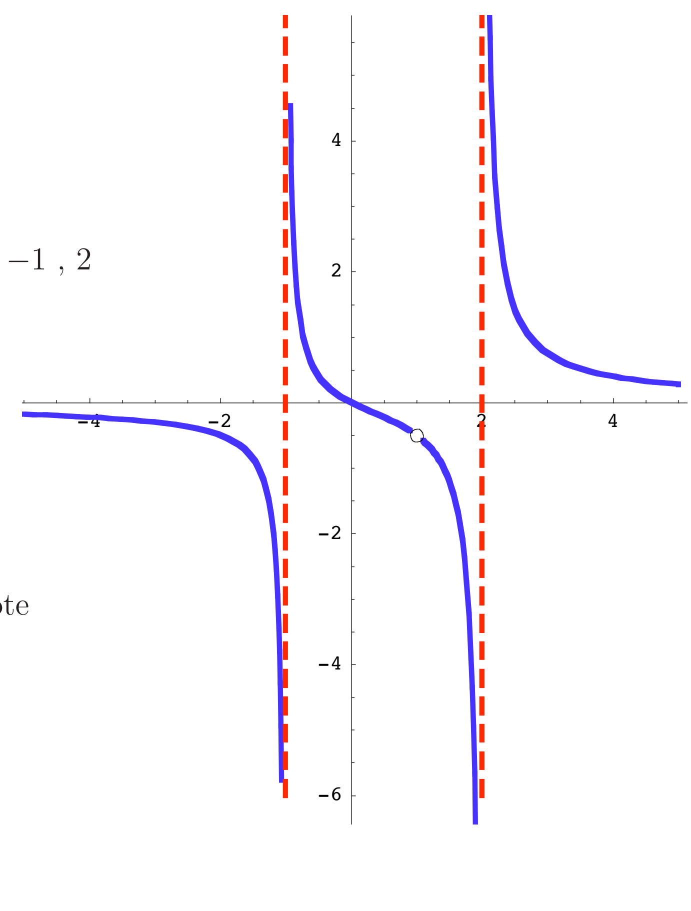
2. Find all **intercepts**, **asymptotes**, and **missing points** (if any) of $y = f(x) = \dfrac{x^2 - x}{(x^2-1)(x-2)}$ .
Then **graph** $f$. (5 pts)

<u>**Ans**</u>: $y = \dfrac{x(x-1)}{(x-1)(x+1)(x-2)} = \dfrac{x}{(x+1)(x-2)}$ , $x \neq 1, -1, 2$

Missing point $= (1, -\frac{1}{2})$
$y = 0 \implies x = 0 \implies$ x–intercept $= (0,0)$
$x = 0 \implies y = 0 \implies$ y–intercept $= (0,0)$
Vertical asymptotes are $x = -1$ , $x = 2$
Horizental asymptote is $y = 0$ (x–axis)

Sign test: zeros of numerator and denominator are $0$ , $-1$ , $2$

| | | -1 | 0 | 2 |
|---|---|---|---|---|
| $x$ | $----$ | $----$ | $++++$ | $++++$ |
| $(x+1)$ | $----$ | $++++$ | $++++$ | $++++$ |
| $(x-2)$ | $----$ | $----$ | $----$ | $++++$ |
| $y$ : | $(-)$ | $(+)$ | $(-)$ | $(+)$ |




Notice that the graph intersects its horizental asymptote at the x–intercept $(0,0)$.

**Math 001.18** **Quiz # 4** **Sem. I , Jan 03 , 2006**

**Name :** ______________________ **ID. # :** ______ **SER. # :** ____

1. Find **all zeros** of the polynomial $p(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$ . (5 pts)

<u>**Ans**</u>:

All coefficients of $p(x)$ are integers.
Factors of $a_0 = -2$ are $\pm 1$ , $\pm 2$
Factors of $a_4 = -2$ are $\pm 1$ , $\pm 3$
Possible rational zeros are $\pm 1$ , $\pm\frac{1}{3}$ , $\pm 2$ , $\pm\frac{2}{3}$

| | | | | | |
|---|---|---|---|---|---|
| $-1$ | 3 | $-4$ | 1 | 6 | $-2$ |
| | | $-3$ | 7 | $-8$ | 2 |
| $\frac{1}{3}$ | 3 | $-7$ | 8 | $-2$ | 0 |
| | | 1 | $-2$ | 2 | |
| | 3 | $-6$ | 6 | 0 | |

So, $-1$ , $\frac{1}{3}$ are rational zeros. To find the other zeros, let $q(x) = 3x^2 - 6x + 6 = 0$

$\Longrightarrow \quad x^2 - 2x + 2 = 0 \quad \Longrightarrow \quad x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$ .

All zeros are $-1$ , $\frac{1}{3}$ , $1+i$ , and $1-i$

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2. Find all **intercepts**, **asymptotes**, and **missing points** (if any) of $y = f(x) = \frac{x^2 - 1}{x^2 + x - 2}$ .
Then **graph** $f$. (5 pts)

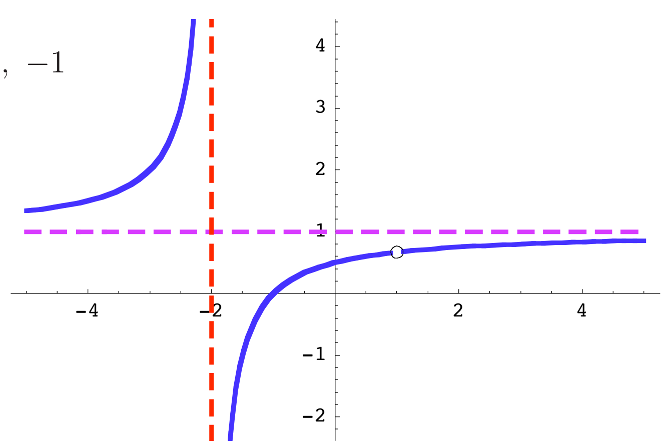
<u>**Ans**</u>: $y = \frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$ , $x \neq -2, 1$

Missing point $= (1, \frac{2}{3})$
$y = 0 \implies x = -1 \implies$ x–intercept $= (-1, 0)$
$x = 0 \implies y = \frac{1}{2} \implies$ y–intercept $= (0, \frac{1}{2})$
Vertical asymptote is $x = -2$
Horizontal asymptote is $y = 1$

Sign test: zeros of numerator and denominator are $-2$ , $-1$

| | | | |
|---|---|---|---|
| $(x+1)$ | $- - - -$ | $- - - -$ | $+ + + +$ |
| $(x+2)$ | $- - - -$ | $+ + + +$ | $+ + + +$ |
| | -2 | -1 | |
| $y$ : | $(+)$ | $(-)$ | $(+)$ |

Let $\frac{x+1}{x+2} = 1 \implies x + 1 = x + 2 \implies 1 = 2$
which is impossible, so the graph does not intersect its horizontal asymptote.



**Math 001.30** **Quiz # 4** **Sem. I , Jan 03 , 2006**

**Name :** ____________________ **ID. # :** ______ **SER. # :** ____

1. Find **all zeros** of the polynomial $p(x) = x^4 - 3x^3 + 5x^2 - x - 10$ . (5 pts)

<u>**Ans**</u>:

All coefficients of $p(x)$ are integers.
Factors of $a_0 = -10$ are $\pm 1$ , $\pm 2$ , $\pm 5$ , $\pm 10$
Factors of $a_4 = 1$ are $\pm 1$
Possible rational zeros are $\pm 1$ , $\pm 2$ , $\pm 5$ , $\pm 10$

| | | | | | |
|---|---|---|---|---|---|
| $-1$ | 1 | $-3$ | 5 | $-1$ | $-10$ |
| | | $-1$ | 4 | $-9$ | 10 |
| 2 | 1 | $-4$ | 9 | $-10$ | 0 |
| | | 2 | $-4$ | 10 | |
| | 1 | $-2$ | 5 | 0 | |

So, $-1$ , 2 are rational zeros. To find the other zeros, let $q(x) = x^2 - 2x + 5 = 0$

$$\Longrightarrow \quad x = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i \ .$$

All zeros are $-1$ , 2 , $1 + 2i$ , and $1 - 2i$

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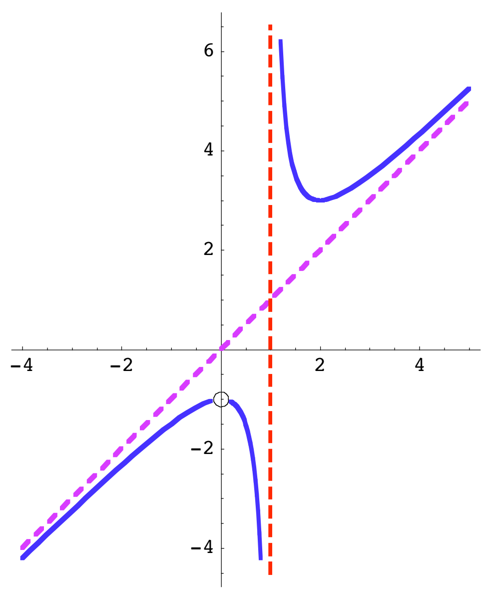
2. Find all **intercepts**, **asymptotes**, and **missing points** (if any) of $y = f(x) = \dfrac{x^3 - x^2 + x}{x^2 - x}$ .
   Then **graph** $f$. (5 pts)

<u>**Ans**</u>: $y = \dfrac{x(x^2 - x + 1)}{x(x-1)} = \dfrac{x^2 - x + 1}{x - 1}$ , $x \neq 0, 1$

Missing point $= (0, -1)$
No x–intercept, because $x^2 - x + 1 \neq 0, \forall x \in \Re$
No y–intercept, because $x \neq 0$
Vertical asymptote is $x = 1$
Slant asymptote is $y = x$ (divide to get it)




Sign test: The only zero of numerator and denominator is 1

| | | |
|---|---|---|
| $(x^2 - x + 1)$ | $+ + + +$ | $+ + + +$ |
| $(x - 1)$ | $- - - -$ | $+ + + +$ |
| | 1 | |
| $y$ : | $(-)$ | $(+)$ |

Let $\dfrac{x^2 - x + 1}{x - 1} = x \implies x^2 - x + 1 = x^2 - x \implies 1 = 0$
which is impossible, so the graph does not intersect its slant asymptote.