Solution of Recitation - R.7 **Done by: A. Al-Shallali**

Q1) (a) $\dfrac{(3\sqrt{x^2-2}-2\sqrt{x^2+2}\,)(3\sqrt{x^2-2}+2\sqrt{x^2+2}\,)}{x\sqrt{5}+\sqrt{26}} = \dfrac{9(x^2-2)-4(x^2+2)}{x\sqrt{5}+\sqrt{26}}$

$$= \frac{5x^2-26}{x\sqrt{5}+\sqrt{26}} = \frac{\cancel{(x\sqrt{5}+\sqrt{26})}(x\sqrt{5}-\sqrt{26})}{\cancel{x\sqrt{5}+\sqrt{26}}} = x\sqrt{5}-\sqrt{26}$$

(b) $= \sqrt{(x+y)^2-4xy} = \sqrt{x^2+2xy+y^2-4xy} = \sqrt{x^2-2xy+y^2} = \sqrt{(x-y)^2} = |x-y|$

Q2) (a) $= \dfrac{3}{\sqrt{5}-\sqrt{2}} \cdot \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} - \dfrac{2}{2\sqrt{5}-3\sqrt{2}} \cdot \dfrac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \dfrac{\cancel{3}\,(\sqrt{5}+\sqrt{2})}{\cancel{(5-2)}} - \dfrac{\cancel{2}\,(2\sqrt{5}+3\sqrt{2})}{\cancel{(20-18)}}$

$$= (\sqrt{5}+\sqrt{2}) - (2\sqrt{5}+3\sqrt{2}) = -\sqrt{5}-2\sqrt{2}$$

(b) $= \dfrac{2}{\sqrt[3]{54}} + \dfrac{4}{\sqrt[3]{16}} - \dfrac{1}{\sqrt[3]{2}} = \dfrac{2}{3\ \sqrt[3]{2}} + \dfrac{4}{2\ \sqrt[3]{2}} - \dfrac{1}{\sqrt[3]{2}} = \dfrac{2}{3\ \sqrt[3]{2}} + \dfrac{2}{\sqrt[3]{2}} - \dfrac{1}{\sqrt[3]{2}}$

$$= \frac{2+6-3}{3\ \sqrt[3]{2}} = \frac{5}{3\ \sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{5\ \sqrt[3]{4}}{3(2)} = \frac{5\ \sqrt[3]{4}}{6}$$

(c) $\dfrac{1}{|\ 2-\sqrt{5}\ |} + \dfrac{1}{|-2-\sqrt{5}\ |} = \dfrac{1}{\sqrt{5}-2} + \dfrac{1}{\sqrt{5}+2} = \dfrac{\sqrt{5}+2+\sqrt{5}-2}{(\sqrt{5}-2)(\sqrt{5}+2)} = \dfrac{2\ \sqrt{5}}{5-4} = 2\ \sqrt{5}$

(d) $\dfrac{\sqrt[13]{(-2)^{13}} + \sqrt[10]{(-2)^{10}}}{\sqrt{2}-1} = \dfrac{(-2)+|-2|}{\sqrt{2}-1} = \dfrac{-2+2}{\sqrt{2}-1} = \dfrac{0}{\sqrt{2}-1} = 0$

Q3) If $x = 7+3\ \sqrt{2}$, $y = 7-3\ \sqrt{2}$, then $x^2+y^2 = (49+42\sqrt{2}+18)+(49-42\sqrt{2}+18) = 134$

which's an integer. So the correct answer is part (e), while the other parts are not integers.

Q4) For any real number a: (a) $\sqrt[4]{a} = |a|$ is true for n = 4 is even ; (b) $\sqrt[4]{a^2} = \sqrt{a}$ is false, because a may be negative (the correction is $\sqrt[4]{a^2} = \sqrt{|a|}$) ; (c) $\sqrt[3]{a} = |a|$ is false, in fact $\sqrt[3]{a} = a$;

(d) $\sqrt[3]{a^2} = \sqrt[6]{a}$ is false, because $\sqrt[3]{a^2} = a^{2/3} \neq a^{1/6} = \sqrt[6]{a}$; (e) $\sqrt[7]{\sqrt[3]{a}} = a^{3/7}$ is false ($\sqrt[7]{\sqrt[3]{a}} = \sqrt[21]{a}$)

Q5) If $x = 2-\sqrt{8}$, then the multiplicative inverse of x is $\dfrac{1}{2-\sqrt{8}} = \dfrac{1}{2-\sqrt{8}} \cdot \dfrac{2+\sqrt{8}}{2+\sqrt{8}} = \dfrac{2+\sqrt{8}}{4-8}$

$$= \frac{2+2\sqrt{2}}{-4} = \frac{2(1+\sqrt{2})}{-4} = -\ \frac{1+\sqrt{2}}{2}$$