Solution of Recitation - 2.2 **Done by: A. Al-Shallali**

Q1) $\frac{1}{3}x^2+2x+\frac{1}{3}y^2-y=0 \implies x^2+6x+y^2-3y=0 \implies (x^2+6x+9)+(y^2-3y+\frac{9}{4})$ $=9+\frac{9}{4} \implies (x+3)^2+(y-\frac{3}{2})^2=\frac{45}{4} \implies \mathrm{C}(-3,\frac{3}{2})\,,\ \mathrm{r}=\frac{\sqrt{45}}{\sqrt{4}}=\frac{3\sqrt{5}}{2}$

Q2) (a) $9x^2+12x+9y^2-18y+13=0 \implies 9(x^2+\frac{4}{3}x+\frac{4}{9})+9(y^2-2y+1)=-13+4+9=0$ $\implies (x+\frac{2}{3})^2+(y-1)^2=0$. This is just the point $(-\frac{2}{3},1)$ (because r = 0)

(b) $x^2+y^2+2x-6y+14=0 \implies (x^2+2x+1)+(y^2-6y+9)=-14+1+9 \implies (x+1)^2+(y-3)^2=-4<0$, so there is no graph.

(c) $x^2-18y-23+12x+y^2=0 \implies (x^2+12x+36)+(y^2-18y+81)=23+36+81=140$ $\implies (x+6)^2+(y-9)^2=140$. It's a circle , $\mathrm{C}(-6,9)\,,\ \mathrm{r}=\sqrt{140}=2\sqrt{35}$

Q3) Let $\mathrm{A}(2,4)$ and $\mathrm{B}(-4,6)$ be the endpoints of a diameter, then the center of the circle will be the midpoint $\implies \mathrm{C}\left(\frac{2-4}{2},\frac{4+6}{2}\right)=\mathrm{C}(-1,5)$ and $\mathrm{r}=\mathrm{d}(\mathrm{A},\mathrm{C})=\sqrt{(2+1)^2+(4-5)^2}$ $=\sqrt{9+1}=\sqrt{10}$. So the equation of the circle is $(x+1)^2+(y-5)^2=10$

Q4) (Extra) : Which one of the following statements is true about the circle $x^2+4x+y^2+6y+4=0$ and the point $\mathrm{A}(-4,-7)$?

(a) A lies on the circle

(b) the circle is tangent to the $y-$axis

(c) A lies inside the circle

(d) the circle is tangent to the $x-$axis

(e) the circle is tangent to both axes

Solution $(x^2+4x+4)+(y^2+6y+9)=-4+4+9 \implies (x+2)^2+(y+3)^2=9 \implies \mathrm{C}(-2,-3)\,,\ \mathrm{r}=3$. Clearly the circle is tangent to the $x-$axis only and the point $\mathrm{A}(-4,-7)$ lies outside the circle, because $\mathrm{d}(\mathrm{A},\mathrm{C})=\sqrt{(-2+4)^2+(-3+7)^2}=\sqrt{20}>\sqrt{9}=3=\mathrm{r}$.