Solution of Recitation - 2.1 **Done by: A. Al-Shallali**

Q1) (I) $A(-2,-5)\,,\ B(1,7)\,,\ C(3,15) \implies d(A,B) = \sqrt{(1+2)^2+(7+5)^2} = \sqrt{9+144} = \sqrt{153}$

$= 3\sqrt{17}\ ,\ d(A,C) = \sqrt{(3+2)^2+(15+5)^2} = \sqrt{25+400} = \sqrt{425} = 5\sqrt{17}\ ,$

$d(B,C) = \sqrt{(3-1)^2+(15-7)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

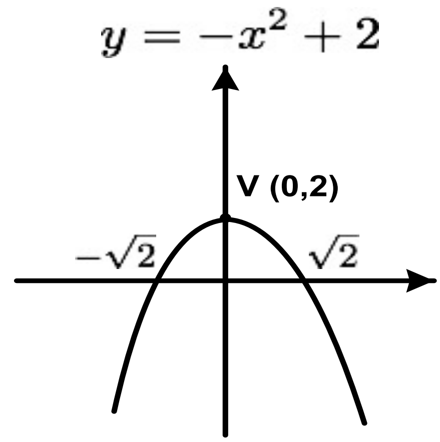
Since $d(A,B)+d(B,C) = 3\sqrt{17}+2\sqrt{17} = 5\sqrt{17} = d(A,C) \implies A\ ,\ B\ ,\ C$ are collinear. While $d^2(A,B)+d^2(B,C) = 153+68 = 221 \neq 425 = d^2(A,C) \implies A\ ,\ B\ ,\ C$ are not vertices of a right triangle.

(II) $A(5,7)\,,\ B(3,9)\,,\ C(6,8) \implies d(A,B) = \sqrt{(5-3)^2+(7-9)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ ,$

$d(A,C) = \sqrt{(6-5)^2+(8-7)^2} = \sqrt{1+1} = \sqrt{2}\ ,\ d(B,C) = \sqrt{(6-3)^2+(8-9)^2} = \sqrt{9+1} = \sqrt{10}$

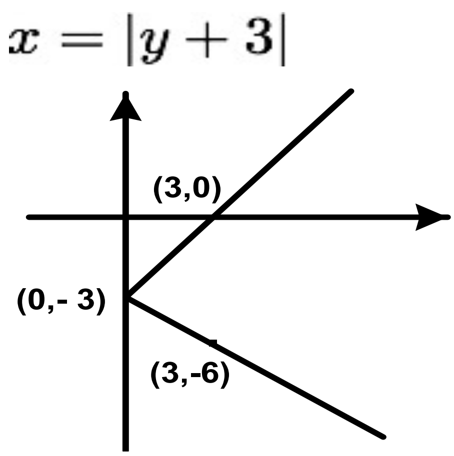
Since $d(A,B)+d(B,C) = 2\sqrt{2}+\sqrt{2} = 3\sqrt{2} \neq \sqrt{10} = d(A,C) \implies A\ ,\ B\ ,\ C$ are not collinear. While $d^2(A,B)+d^2(B,C) = 8+2 = 10 = d^2(A,C) \implies A\ ,\ B\ ,\ C$ are vertices of a right triangle.

Q2) (a) The graph of $y = -x^2+2$ is a parabola opens downward with vertex $V(0,2)$. If $y=0$, then $x^2 = 2 \implies x = \pm\sqrt{2} = x-$intecepts.



(b) $x = |y+3| = \begin{cases} y+3 & \text{if } \ y \geq -3 \\ -y-3 & \text{if } \ y < -3 \end{cases}$

The graph consists of two half lines L_1 , L_2, where $(0,-3)\ ,\ (0,3) \in L_1$ and $(0,-3)\ ,\ (3,-6) \in L_2$. Notice that $x \geq 0$



Q3) $A(x,-9)\,,\ B(3,-5)\,,\ d(A,B) = 6 \implies \sqrt{(x-3)^2+(-9+5)^2} = 6 \implies (x-3)^2+16 = 36$

$\implies (x-3)^2 = 20 \implies x-3 = \pm\sqrt{20} = \pm 2\sqrt{5} \implies x = 3 \pm 2\sqrt{5}$ (please turn on)

Q4) Midpoint $= \mathrm{M}(4,6) = \mathrm{M}\left(\dfrac{x/2\,+\,3x/2}{2}\,,\,\dfrac{y\,+\,y/2}{2}\right) = \mathrm{M}\left(\dfrac{2x}{2}\,,\,\dfrac{3y/2}{2}\right) = \mathrm{M}\left(x\,,\,\dfrac{3y}{4}\right)$

$\Longrightarrow \boxed{x=4}$ and $6 = \dfrac{3y}{4} \Longrightarrow 3y = 24 \Longrightarrow \boxed{y=8}$

Q5) (Extra) : Find all points (a,b) that are 4 units from the origin and $2\sqrt{6}$ units from the point $(2,0)$.

<u>Solution</u> $\mathrm{P}(a,b)$, $\mathrm{O}(0,0)$, $\mathrm{A}(2,0)$, where $\mathrm{d(P,O)} = 4$ and $\mathrm{d(P,A)} = 2\sqrt{6}$

$\Longrightarrow \sqrt{(a-0)^2+(b-0)^2} = 4 \Longrightarrow \boxed{a^2+b^2=16}$ and

$\sqrt{(a-2)^2+(b-0)^2} = 2\sqrt{6} \Longrightarrow (a-2)^2+b^2 = 24 \Longrightarrow (a^2+b^2)-4a+4 = 24$

$\Longrightarrow 16-4a+4 = 24 \Longrightarrow -4a = 4 \Longrightarrow a = -1$, but $b^2 = 16-a^2 = 16-1 = 15 \Longrightarrow b = \pm\sqrt{15}$

Thus the points are $(-1,-\sqrt{15})$, $(-1,\sqrt{15})$