

Solution of Recitation - 1.8

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Q1) $\left| \frac{2x+5}{3} \right| < \frac{3}{4} + \frac{1}{2} = \frac{5}{4} \Rightarrow -\frac{5}{4} < \frac{2x+5}{3} < \frac{5}{4}$ (multiply by 12)

$$-15 < 8x + 20 < 15 \Rightarrow -35 < 8x < -5 \Rightarrow -\frac{35}{8} < x < -\frac{5}{8} \Rightarrow \text{S.S.} = \left(-\frac{35}{8}, -\frac{5}{8} \right)$$

Q2) $\frac{-7 + 5|3x-4|}{7|3x-4|-2} = -3 \Rightarrow -7 + 5|3x-4| = -21|3x-4| + 6 \Rightarrow 26|3x-4| = 13$

$$\Rightarrow |3x-4| = \frac{1}{2} \Rightarrow 3x-4 = -\frac{1}{2} \text{ or } 3x-4 = \frac{1}{2} \Rightarrow 3x = \frac{7}{2} \text{ or } 3x = \frac{9}{2}$$

$$\Rightarrow x = \frac{7}{6} \text{ or } x = \frac{3}{2} \Rightarrow \text{S.S.} = \left\{ \frac{7}{6}, \frac{3}{2} \right\}$$

Q3) $|2x+8|^2 - |9x+36| \geq 9 \Rightarrow 4|x+4|^2 - 9|x+4| - 9 \geq 0.$

Let $y = |x+4|$, so the equation is $4y^2 - 9y - 9 \geq 0$

$$\Rightarrow (4y+3)(y-3) \geq 0 \Rightarrow (\text{from sign test}) y \leq -\frac{3}{4} \text{ or } y \geq 3$$

$$\Rightarrow |x+4| \leq -\frac{3}{4} \text{ (impossible)} \text{ or } |x+4| \geq 3 \Rightarrow x+4 \leq -3$$

$$\text{or } x+4 \geq 3 \Rightarrow x \leq -7 \text{ or } x \geq -1$$

$$\Rightarrow \text{S.S.} = (-\infty, -7] \cup [-1, \infty)$$

$$\begin{array}{r} (4y+3) \quad \text{---} \quad |++|++ \\ (y-3) \quad \text{---} \quad |- -|++ \\ \hline \end{array}$$

L. S. is : + -3/4 - 3 +

Q4) $|5x-1| = |2x+3| \Rightarrow 5x-1 = 2x+3 \text{ or } 5x-1 = -(2x+3) = -2x-3 \Rightarrow 3x = 4$

$$\text{or } 7x = -2 \Rightarrow x = \frac{4}{3} \text{ or } x = -\frac{2}{7}.$$

Thus the solution set contains one positive and one negative rational number.

Q5) (Extra) : Solve the equation $4 \leq (x+3)^2 \leq 9$

$$\Rightarrow 2 \leq |x+3| \leq 3 \Rightarrow |x+3| \geq 2 \text{ and } |x+3| \leq 3 \Rightarrow (x+3 \leq -2 \text{ or } x+3 \geq 2) \text{ and } (-3 \leq x+3 \leq 3) \Rightarrow S_1 : (x \leq -5 \text{ or } x \geq -1) \text{ and } S_2 : (-6 \leq x \leq 0).$$

$$\text{S.S.} = S_1 \cap S_2 = \left[(-\infty, -5] \cup [-1, \infty) \right] \cap [-6, 0] = [-6, -5] \cup [-1, 0]$$