Solution of Recitation - 1.7 **Done by: A. Al-Shallali**

Q1) (a) $-2 < \dfrac{2x-3}{3} \leq 1 \implies -6 < 2x-3 \leq 3 \implies -3 < 2x \leq 6 \implies -\dfrac{3}{2} < x \leq 3$

$\implies$ S.S. $= \left(-\dfrac{3}{2}, 3\right]$

(b) $\dfrac{(2-x)\,(x+3)^4}{(x-5)^3} \leq 0\ ,\ x \neq 5 \implies$ S.S. $= (-\infty, 2] \cup (5, \infty)$

$(2-x)$	$++$	$++$	$--$	$--$
$(x+3)^4$	$++$	$++$	$++$	$++$
$(x-5)^3$	$--$	$--$	$--$	$++$

L.S. is : $-$ -3 $-$ 2 $+$ 5 $-$

(c) $\dfrac{3x+1}{2x-3} < 4 \implies \dfrac{3x+1}{2x-3} - 4 < 0 \implies \dfrac{3x+1-8x+12}{2x-3} < 0$

$\implies \dfrac{13-5x}{2x-3} < 0 \implies$ S.S. $= \left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{13}{5}, \infty\right)$

(13-5x)	$++$	$++$	$--$
(2x-3)	$--$	$++$	$++$

L. S. is : $-$ 3/2 $+$ 13/5 $-$

Q2) The equation $2x^2 - \sqrt{3}x + 2k - \dfrac{1}{4} = 0$ has no real solution when $b^2 - 4ac < 0$

$\implies 3 - 4(2)(2k - 1/4) < 0 \implies 3 - 16k + 2 < 0 \implies 5 < 16k \implies k > \dfrac{5}{16} \implies k \in \left(\dfrac{5}{16}, \infty\right)$

Q3) $\dfrac{1}{x^2+2x-3} \leq \dfrac{3}{x+3} \implies \dfrac{1}{(x+3)(x-1)} - \dfrac{3}{x+3} \leq 0$

$\implies \dfrac{1-3x+3}{(x+3)(x-1)} \leq 0 \implies \dfrac{4-3x}{(x+3)(x-1)} \leq 0$

$\implies$ S.S. $= (-3, 1) \cup \left[\dfrac{4}{3}, \infty\right)$

(4 - 3x)	$++$	$++$	$++$	$--$
(x + 3)	$--$	$++$	$++$	$++$
(x - 1)	$--$	$--$	$++$	$++$

L.S. is : $+$ -3 $-$ 1 $+$ 4/3 $-$

Q4) $(2x+1)^2 < 3(1-x) \implies 4x^2 + 4x + 1 - 3 + 3x < 0$

$\implies 4x^2 + 7x - 2 < 0 \implies (4x-1)(x+2) < 0 \implies$ S.S. $= \left(-2, \dfrac{1}{4}\right)$

(4x - 1)	$--$	$--$	$++$
(x + 2)	$--$	$++$	$++$

L. S. is : $+$ -2 $-$ 1/4 $+$

Q5) (Extra) : Find all values of k for which the equation $kx^2 - 4x + k = 3$ has

(a) only one real root (b) two different real roots (c) no real roots (2 non-real comlex roots)

Solution (a) $kx^2 - 4x + (k-3) = 0$, so $a = k$, $b = -4$, $c = k - 3$. In this case $b^2 - 4ac = 0$

$\implies 16 - 4k(k-3) = 0 \implies 4 - k(k-3) = 0 \implies 4 - k^2 + 3k = 0 \implies k^2 - 3k - 4 = 0 \implies (k-4)(k+1) = 0$

$\implies k = 4$ or $k = -1$

(please turn on)

(b) $b^2 - 4ac > 0 \implies 16 - 4k(k-3) > 0 \implies 4 - k^2 + 3k > 0$

$\implies k^2 - 3k - 4 < 0 \implies (k-4)(k+1) < 0 \implies k \in (-1, 4)$

(k+1)	- -	+ +	+ +
(k-4)	- -	- -	+ +
	-1	4	
L. S. is :	+	-	+

(c) $b^2 - 4ac < 0 \implies 16 - 4k(k-3) < 0 \implies 4 - k^2 + 3k < 0 \implies k^2 - 3k - 4 > 0 \implies (k-4)(k+1) > 0$

(from the above figure) $k \in (-\infty, -1) \cup (4, \infty)$