Solution of Recitation - 1.6 **Done by: A. Al-Shallali**

Q1) (a) $\dfrac{x}{x+5} - \dfrac{5}{5-x} = \dfrac{50}{x^2-25} \implies \dfrac{x}{x+5} + \dfrac{5}{x-5} = \dfrac{50}{(x+5)(x-5)}$, $x \neq -5$, $x \neq 5$

Multiply by LCD $= (x+5)(x-5) \implies x(x-5) + 5(x-5) = 50 \implies x^2 = 25 \implies x = \pm 5$

(but both values are rejected), therefore S.S. $= \phi$

(b) $\sqrt[4]{x^4 + x^2 + 2x} = x \implies x^4 + x^2 + 2x = x^4 \implies x^2 + 2x = 0 \implies x(x+2) = 0 \implies x = 0$ or $x = -2$. We must check: For $x = 0$, L.S. $= \sqrt[4]{0} = 0 =$ R.S. and for $x = -2$, L.S. $= \sqrt[4]{16} = 2 \neq$ R.S. $= -2$. Thus S.S. $= \{0\}$.

(c) $x^{2/3} + 7x^{1/3} = 8$, let $y = x^{1/3}$, so $y^2 + 7y - 8 = 0 \implies (y-1)(y+8) = 0 \implies y = 1$ or $y = -8$

$\implies x^{1/3} = 1$ or $x^{1/3} = -8 \implies x = 1$ or $x = -512 \implies$ S.S. $= \{-512, 1\}$.

Q2) To find the number of real solutions of the equation $\dfrac{5}{(1-x^2)^2} + \dfrac{7}{1-x^2} = 6$, let $y = \dfrac{1}{1-x^2}$,

$x \neq \pm 1$, so $5y^2 + 7y - 6 = 0 \implies (5y-3)(y+2) = 0 \implies y = \dfrac{3}{5}$ or $y = -2$

$\implies \dfrac{1}{1-x^2} = \dfrac{3}{5} \implies 3 - 3x^2 = 5 \implies 3x^2 = -2$ (impossible) or $\dfrac{1}{1-x^2} = -2$

$\implies 1 = 2x^2 - 2 \implies 2x^2 = 3 \implies x^2 = \dfrac{3}{2} \implies x = \pm\sqrt{3/2}$

This means that the equation has two real solutions.

Q3) To find the sum of all real solutions of the equation $(2t-1)^{2/3} + (16t-8)^{1/3} = 3$

$\implies (2t-1)^{2/3} + 2(2t-1)^{1/3} - 3 = 0$, let $y = (2t-1)^{1/3} \implies y^2 + 2y - 3 = 0 \implies (y-1)(y+3) = 0$

$\implies y = 1$ or $y = -3 \implies (2t-1)^{1/3} = 1$ or $(2t-1)^{1/3} = -3 \implies 2t - 1 = 1$ or $2t - 1 = -27$

$\implies 2t = 2$ or $2t = -26 \implies t = 1$ or $t = -13$. Therefore sum of all real solutions $= 1 + (-13) = -12$

Q4) $\sqrt{2x} = \sqrt{x+7} - 1 \implies \sqrt{2x} + 1 = \sqrt{x+7} \implies 2x + 2\sqrt{2x} + 1 = x + 7 \implies 2\sqrt{2x} = 6 - x$

$\implies 4(2x) = 36 - 12x + x^2 \implies x^2 - 20x + 36 = 0 \implies (x-2)(x-18) = 0 \implies x = 2$ or $x = 18$.

Again, we must check: for $x = 2$, L.S. $= \sqrt{4} = 2 =$ R.S. $= \sqrt{9} - 1$ and for $x = 18$, L.S. $= \sqrt{36} = 6$ $\neq$ R.S. $= \sqrt{25} - 1 \implies$ S.S. $= \{2\}$. Thus the solution set consists of only one even positive integer.

Q5) (Extra) Find all real solutions of the equation $\sqrt{2x+3} + \sqrt{-x-5} + 4 = 0$

__**Solution**__ $\implies \sqrt{2x+3} + \sqrt{-x-5} = -4$, clearly and without any work, the equation has no real solution, because R.S. is negative while L.S. is never negative. So S.S. $= \phi$