Solution of Recitation - 1.4 **Done by: A. Al-Shallali**

Q1) The quadratic equation $x^2 + 11x + k^2 = 0$ has exactly one real solution when $b^2 - 4ac = 0$

$\Longrightarrow 121 - 4k^2 = 0 \Longrightarrow k^2 = \dfrac{121}{4} \Longrightarrow k = \pm\dfrac{11}{2}$

Q2) $-\dfrac{1}{2}$ satisfies the equation $(2x-1)(3x+2) = k$, so $(-1-1)(-3/2+2) = k \Longrightarrow k = (-2)(1/2) = -1$.

The equation is $(2x-1)(3x+2) = -1 \Longrightarrow 6x^2 + x - 1 = 0 \Longrightarrow (3x-1)(2x+1) = 0$

$\Longrightarrow x = \dfrac{1}{3}$ or $x = -\dfrac{1}{2}$. Therefore the other solution is $\dfrac{1}{3}$.

Q3) To write the equation $2x^2 - 8x - 5 = 0$ in the form $(x-a)^2 = b$, we must complete the square :

$x^2 - 4x = \dfrac{5}{2} \Longrightarrow x^2 - 4x + 4 = \dfrac{5}{2} + 4 \Longrightarrow (x-2)^2 = \dfrac{13}{2} \Longrightarrow a = 2$ and $b = \dfrac{13}{2}$.

Q4) $a^2x^2 - 2abx + b^2 - 16 = 0 \Longrightarrow$ Disc. $= \mathrm{B}^2 - 4\mathrm{AC} = (4\mathrm{a}^2\mathrm{b}^2) - 4(\mathrm{a}^2)(\mathrm{b}^2 - 16) =$

$4\mathrm{a}^2\mathrm{b}^2 - 4\mathrm{a}^2\mathrm{b}^2 + 64\mathrm{a}^2 = 64\mathrm{a}^2 \Longrightarrow x = \dfrac{-\mathrm{B} \pm \sqrt{64\mathrm{a}^2}}{2\mathrm{A}} = \dfrac{2\mathrm{ab} \pm 8|\mathrm{a}|}{2\mathrm{a}^2} = \dfrac{2\mathrm{ab} \pm 8\mathrm{a}}{2\mathrm{a}^2} = \dfrac{\mathrm{b} \pm 4}{\mathrm{a}}$.

Q5) Let x_1 , x_2 , where $x_1 < x_2$ be the solutions of the equation $x(6x+7) = 3$

$\Longrightarrow 6x^2 + 7x - 3 = 0 \Longrightarrow (3x-1)(2x+3) = 0 \Longrightarrow x = \dfrac{1}{3}$ or $x = -\dfrac{3}{2}$

$\Longrightarrow 2x_1 + 3x_2 = 2(-3/2) + 3(1/3) = -3 + 1 = -2$

Q6) (Extra) If r_1 , r_2 are the roots of the equation $x^2 + x + 2 = 0$, then find the quadratic equation whose roots are $\dfrac{1}{r_1}$, $\dfrac{1}{r_2}$.

__Solution__: $r_1 + r_2 = -\dfrac{b}{a} = -1$ and $r_1 \cdot r_2 = \dfrac{c}{a} = 2$. The quadratic equation whose roots are $\dfrac{1}{r_1}$, $\dfrac{1}{r_2}$

is $x^2 - \left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)x + \dfrac{1}{r_1} \cdot \dfrac{1}{r_2} = 0 \Longrightarrow x^2 - \left(\dfrac{r_2 + r_1}{r_1 \cdot r_2}\right)x + \dfrac{1}{r_1 \cdot r_2} = 0$

$\Longrightarrow x^2 - \dfrac{-1}{2}x + \dfrac{1}{2} = 0 \Longrightarrow 2x^2 + x + 1 = 0$