Math 001–Some Properties Of Graphs

Horizontal and Vertical Translation					
$y = \boldsymbol{f(x-2)}$	$y = \boldsymbol{f(x+2)}$	$y = \boldsymbol{f(x)+1}$	$y = \boldsymbol{f(x)-1}$	$y = \boldsymbol{f(x+2)+1}$	$y = \boldsymbol{f(x-2)-1}$
To get the graph of each of the above functions, just shift the graph of $y = f(x)$					
horizontally 2 units to the right	horizontally 2 units to the left	vertically 1 unit upward	vertically 1 unit downward	hor. 2 units to the left and ver. 1 unit upward	hor. 2 units to the right and ver. 1 unit downward
If the point (a,b) lies on the graph of $y = f(x)$, then the following points lie on the graph of the above functions					
$\boldsymbol{(a+2,b)}$	$\boldsymbol{(a-2,b)}$	$\boldsymbol{(a,b+1)}$	$\boldsymbol{(a,b-1)}$	$\boldsymbol{(a-2,b+1)}$	$\boldsymbol{(a+2,b-1)}$
To find the equation of the above functions from the equation of $y = f(x)$, just replace					
$\boldsymbol{x}$ by $\boldsymbol{x-2}$	$\boldsymbol{x}$ by $\boldsymbol{x+2}$	$\boldsymbol{y}$ by $\boldsymbol{y-1}$	$\boldsymbol{y}$ by $\boldsymbol{y+1}$	$\boldsymbol{x}$ by $\boldsymbol{x+2}$ and $\boldsymbol{y}$ by $\boldsymbol{y-1}$	$\boldsymbol{x}$ by $\boldsymbol{x-2}$ and $\boldsymbol{y}$ by $\boldsymbol{y+1}$

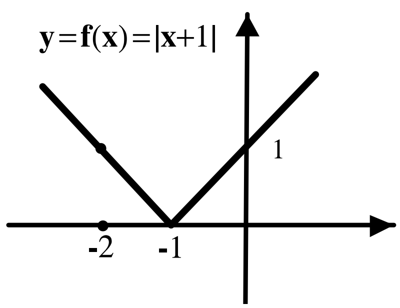
<u>Problem 1</u>: If the graph of the equation $y = 2x^2 - 4x$ is shifted horizontally one unit to the left and vertically two units upward, then find the equation of the new graph.

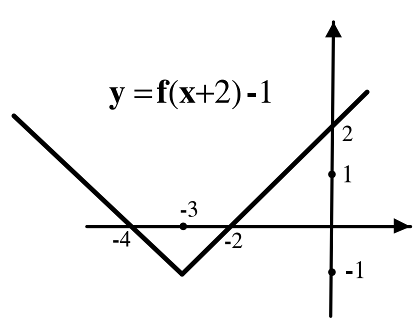
<u>Solution</u>: Replce x by $(x+1)$ and y by $(y-2)$ in the original equation, to get
$y - 2 = 2(x+1)^2 - 4(x+1) \implies y = 2(x^2 + 2x + 1) - 4x - 4 + 2 = 2x^2 + 4x + 2 - 4x - 2 = 2x^2$.
So the new equation is $y = 2x^2$

<u>Problem 2</u>: (a) Use the graph of $y = f(x) = |x+1|$ to graph $y = f(x+2) - 1$

(b) Write the equation of $y = f(x+2) - 1$.

<u>Solution</u>: (a) Shift the graph of $y = f(x) = |x+1|$ 2 units to the left and one unit downward to get $y = f(x+2) - 1$. So the point $(-1,0)$ on the graph of $y = f(x)$ will be shifted to the point $(-1-2, 0-1) = (-3,-1)$ on the graph of $y = f(x+2) - 1$. Similarly, $(0,1) \longrightarrow (-2,0)$; $(-2,1) \longrightarrow (-4,0)$ and $(2,3) \longrightarrow (0,2)$





(b) To find the equation of the second graph, replace x by $x+2$ and y by $y+1$ to get $y + 1 = |(x+2) + 1| \implies y = |x+3| - 1$

	Reflection Across			Compression And Stretching		
	$y-$ axis	$x-$ axis	Both axes	Horizontal	Vertical	Both
Point On $\boldsymbol{y=f(x)}$	Point On $\boldsymbol{y=f(-x)}$	Point On $\boldsymbol{y=-f(x)}$	Point On $\boldsymbol{y=-f(-x)}$	Point On $\boldsymbol{y=f(mx)}$	Point On $\boldsymbol{y=kf(x)}$	Point On $\boldsymbol{y=kf(mx)}$
$\boldsymbol{(a,b)}$	$\boldsymbol{(-a,b)}$	$\boldsymbol{(a,-b)}$	$\boldsymbol{(-a,-b)}$	$\boldsymbol{\left(\frac{a}{m},b\right)}$	$\boldsymbol{(a,kb)}$	$\boldsymbol{\left(\frac{a}{m},kb\right)}$
$\boldsymbol{(-2,3)}$	$\boldsymbol{(2,3)}$	$\boldsymbol{(-2,-3)}$	$\boldsymbol{(2,-3)}$	$\boldsymbol{\left(-\frac{2}{m},3\right)}$	$\boldsymbol{(-2,3k)}$	$\boldsymbol{\left(-\frac{2}{m},3k\right)}$

<u>Complex Examples</u>

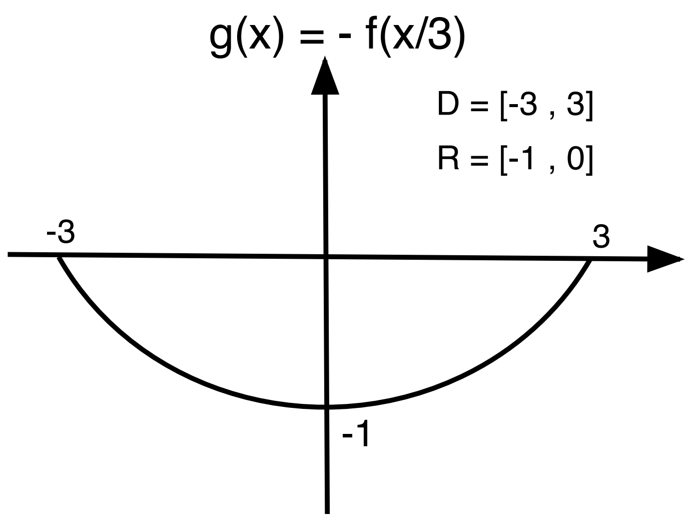
Point On $\boldsymbol{y=f(x)}$	Point On $\boldsymbol{y=2f(3x)}-1$	Point On $\boldsymbol{y=-3f\left(\frac{x}{2}\right)}+2$	Point On $\boldsymbol{y=\frac{1}{2}f\left(-\frac{x}{4}\right)}-5$
$\boldsymbol{(a,b)}$	$\boldsymbol{\left(\frac{a}{3},2b-1\right)}$	$\boldsymbol{(2a,-3b+2)}$	$\boldsymbol{\left(-4a,\frac{1}{2}b-5\right)}$
$\boldsymbol{(3,-1)}$	$\boldsymbol{(1,-3)}$	$\boldsymbol{(6,5)}$	$\boldsymbol{\left(-12,-\frac{11}{2}\right)}$

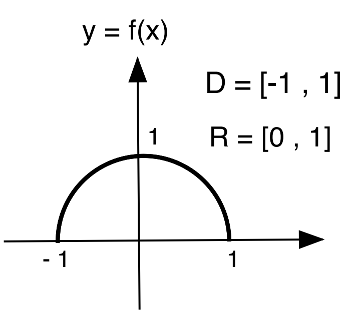
<u>Problem 3</u>: If $f(x)$ is an **odd** function such that $f(2)=1$, find two points on the graph of the function $g(x)=2f(3x)-1$

<u>Solution</u>: Since $f(x)$ is odd, then $f(-2)=-f(2)=-1$. Therefore, the points $(2,1)$ and $(-2,-1)$ lie on the graph of $y=f(x)$. From the above tables: the points $\left(\frac{2}{3},2(1)-1\right)=\left(\frac{2}{3},1\right)$ and $\left(-\frac{2}{3},2(-1)-1\right)=\left(-\frac{2}{3},-3\right)$ lie on the graph of the $g(x)=2f(3x)-1$.

<u>Problem 4</u>: Use the graph of $y=f(x)=\sqrt{1-x^2}$ to graph $g(x)=-f\left(\frac{x}{3}\right)$

<u>Solution</u>: The graph of $f(x)$ is the upper half of a circle of center $C(0,0)$ and radius $=1$. To graph $g(x)$, first stretch the graph of $f(x)$ horizontally by a factor of $\frac{1}{1/3}=3$, then reflect the new graph across the x–axix (the graph of $g(x)$ is a lower half of an ellipse)





Instructor: A. Al-Shallali