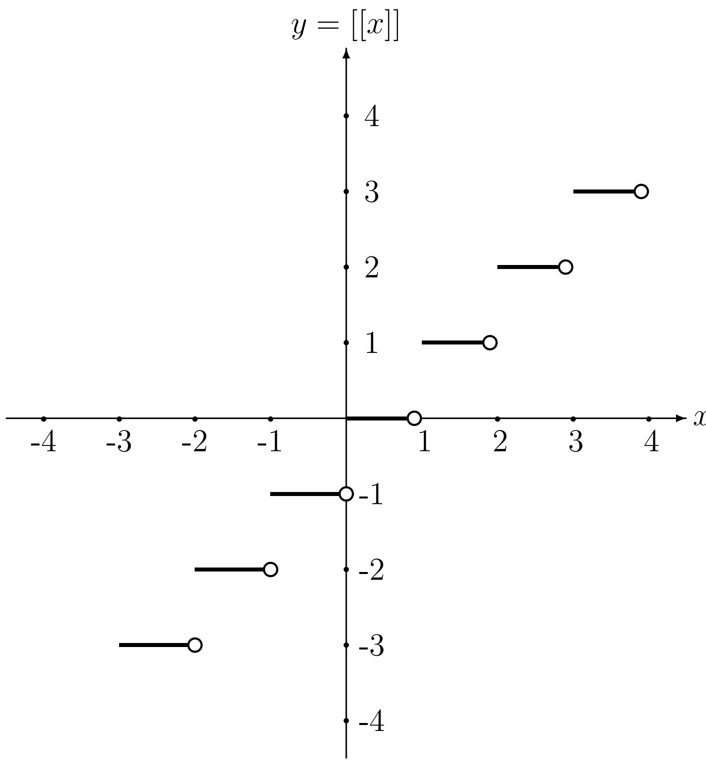
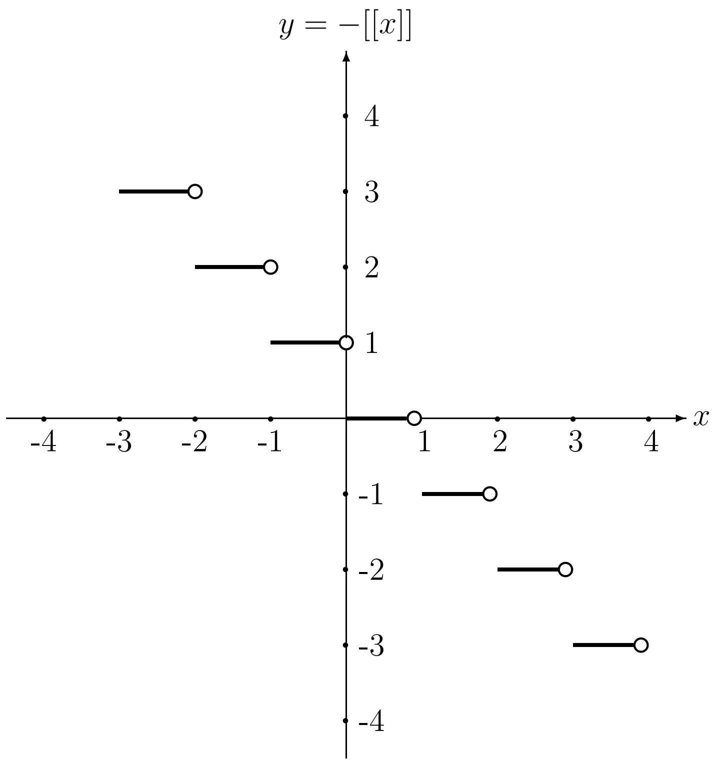
# Math 001 – Greatest Integer Function   By: A. Al-Shallali

**Definition**: The greatest integer function $y = [[x]]$ is the greatest integer less than or equal to $x$.

**Example**: $[[0]] = [[0.01]] = [[0.5]] = [[.9]] = [[0.99]] = 0$, $[[1]] = [[1.4]] = [[1.98]] = 1$, $[[-1]] = [[-.95]] = [[-.01]] = -1$, $[[7.8]] = 7$, $[[-39.6]] = -40$, $[[\pi]] = 3$, $[[-\sqrt{2}]] = -2$, $[[-\sqrt{8}]] = -3$




| $y = [[x]]$ | |
|---|---|
| $x$ | $y = [[x]]$ |
| $\vdots$ | $\vdots$ |
| $-3 \le x < -2$ | $-3$ |
| $-2 \le x < -1$ | $-2$ |
| $-1 \le x < 0$ | $-1$ |
| $0 \le x < 1$ | $0$ |
| $1 \le x < 2$ | $1$ |
| $2 \le x < 3$ | $2$ |
| $3 \le x < 4$ | $3$ |
| $\vdots$ | $\vdots$ |




| $y = -[[x]]$ | | |
|---|---|---|
| $x$ | $[[x]]$ | $y = -[[x]]$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3 \le x < -2$ | $-3$ | $3$ |
| $-2 \le x < -1$ | $-2$ | $2$ |
| $-1 \le x < 0$ | $-1$ | $1$ |
| $0 \le x < 1$ | $0$ | $0$ |
| $1 \le x < 2$ | $1$ | $-1$ |
| $2 \le x < 3$ | $2$ | $-2$ |
| $3 \le x < 4$ | $3$ | $-3$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Note: This graph can be obtained by reflecting the graph of y=[[x]] about the x–axix. | | |



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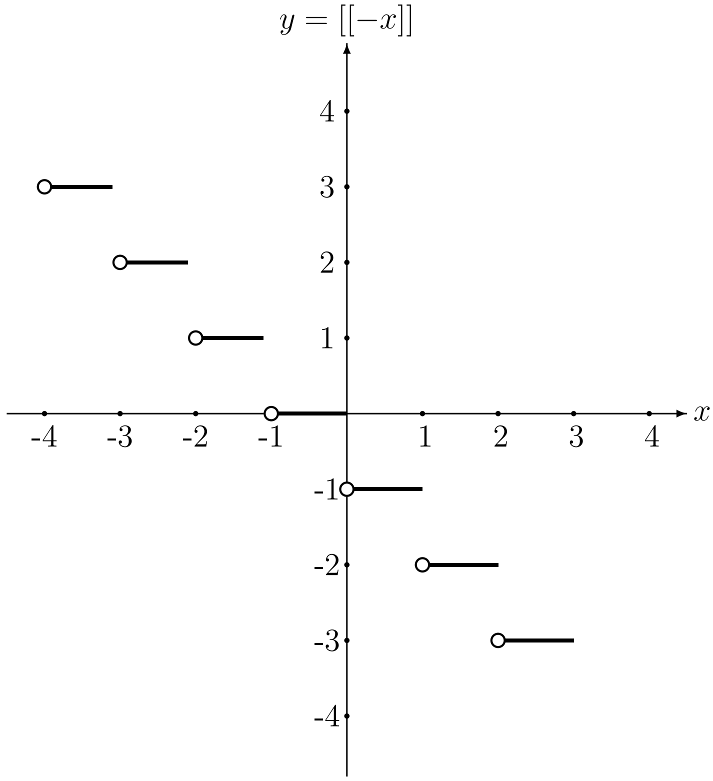
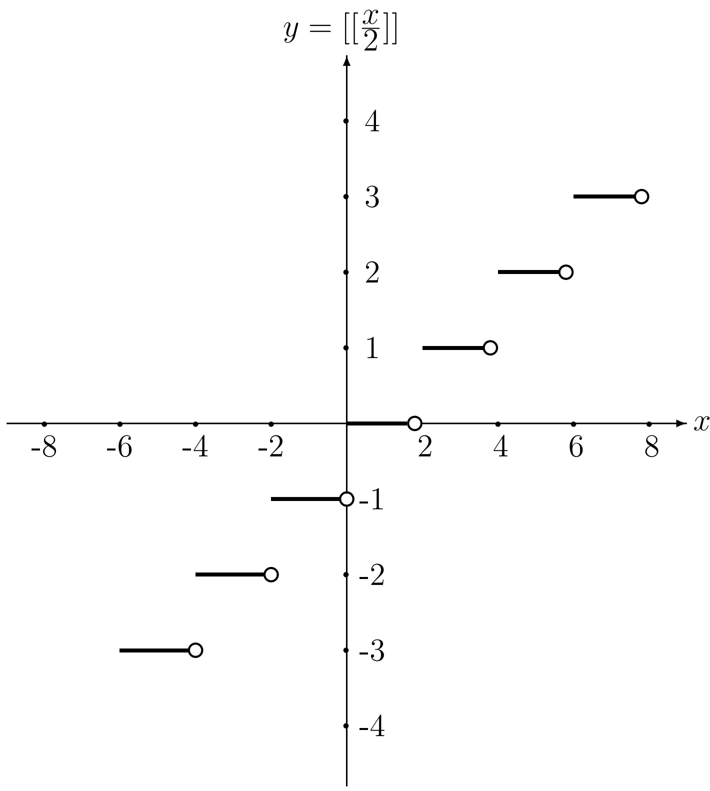
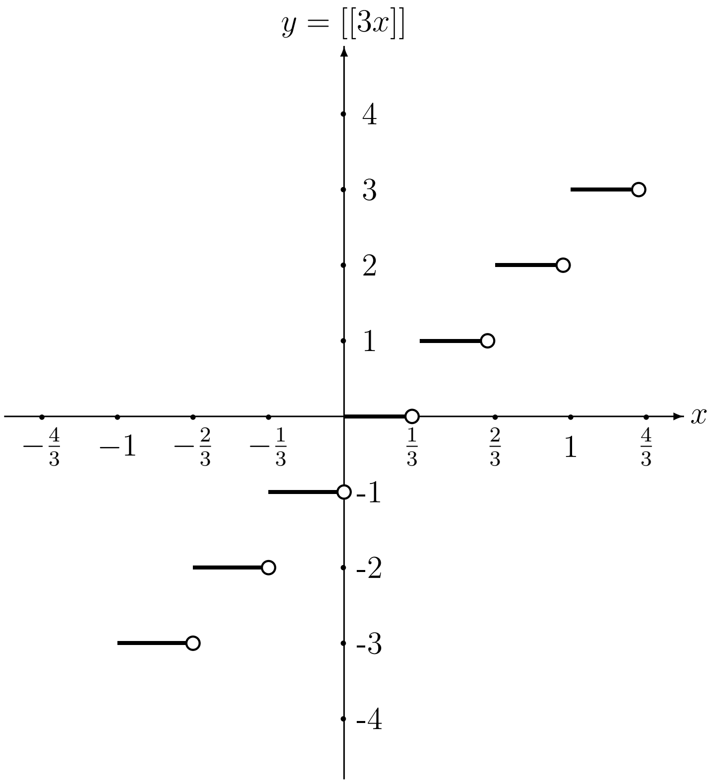
| $y=[[-x]]$ | | |
|---|---|---|
| $-x$ | $x$ | $y=[[-x]]$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3\le -x<-2$ | $3\ge x>2$ | $-3$ |
| $-2\le -x<-1$ | $2\ge x>1$ | $-2$ |
| $-1\le -x<0$ | $1\ge x>0$ | $-1$ |
| $0\le -x<1$ | $0\ge x>-1$ | $0$ |
| $1\le -x<2$ | $-1\ge x>-2$ | $1$ |
| $2\le -x<3$ | $-2\ge x>-3$ | $2$ |
| $3\le -x<4$ | $-3\ge x>-4$ | $3$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Note: This graph can be obtained by reflecting the graph of y=[[x]] about the y–axix. | | |



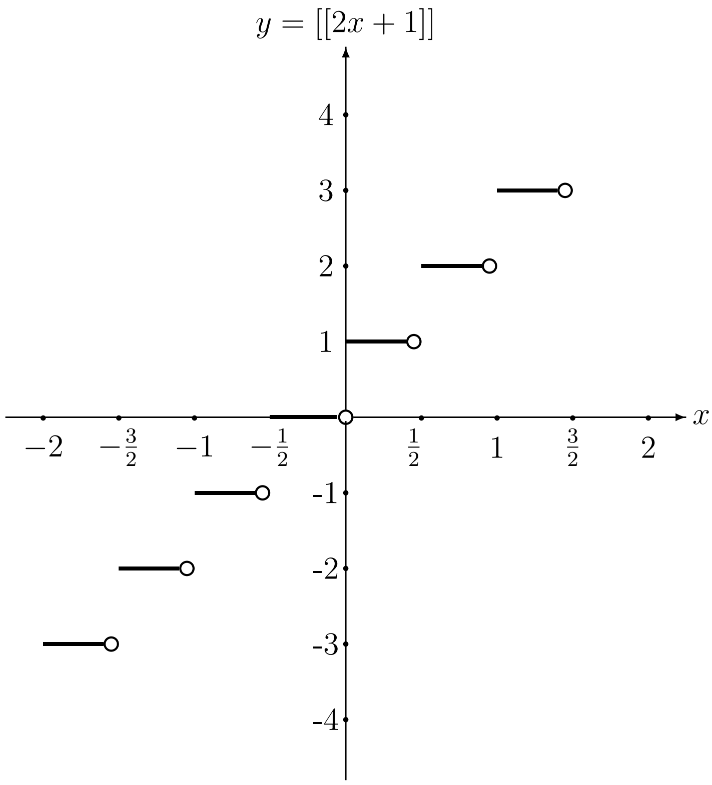

| $y=[[\frac{x}{2}]]$ | | |
|---|---|---|
| $\frac{x}{2}$ | $x$ | $y=[[\frac{x}{2}]]$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3\le \frac{x}{2}<-2$ | $-6\le x<-4$ | $-3$ |
| $-2\le \frac{x}{2}<-1$ | $-4\le x<-2$ | $-2$ |
| $-1\le \frac{x}{2}<0$ | $-2\le x<0$ | $-1$ |
| $0\le \frac{x}{2}<1$ | $0\le x<2$ | $0$ |
| $1\le \frac{x}{2}<2$ | $2\le x<4$ | $1$ |
| $2\le \frac{x}{2}<3$ | $4\le x<6$ | $2$ |
| $3\le \frac{x}{2}<4$ | $6\le x<8$ | $3$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Note: This graph can be obtained by stretching the graph of y=[[x]] horizentally by a factor of 2. | | |



| $y = [[3x]]$ | | |
|---|---|---|
| $3x$ | $x$ | $y = [[3x]]$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3 \le 3x < -2$ | $-1 \le x < -\frac{2}{3}$ | $-3$ |
| $-2 \le 3x < -1$ | $-\frac{2}{3} \le x < -\frac{1}{3}$ | $-2$ |
| $-1 \le 3x < 0$ | $-\frac{1}{3} \le x < 0$ | $-1$ |
| $0 \le 3x < 1$ | $0 \le x < \frac{1}{3}$ | $0$ |
| $1 \le 3x < 2$ | $\frac{1}{3} \le x < \frac{2}{3}$ | $1$ |
| $2 \le 3x < 3$ | $\frac{2}{3} \le x < 1$ | $2$ |
| $3 \le 3x < 4$ | $1 \le x < \frac{4}{3}$ | $3$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Note: This graph can be obtained by shrinking the graph of y=[[x]] horizontally by a factor of $\frac{1}{3}$. | | |




| $y = [[2x+1]]$ | | |
|---|---|---|
| $2x+1$ | $x$ | $y = [[2x+1]]$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3 \le 2x+1 < -2$ | $-2 \le x < -\frac{3}{2}$ | $-3$ |
| $-2 \le 2x+1 < -1$ | $-\frac{3}{2} \le x < -1$ | $-2$ |
| $-1 \le 2x+1 < 0$ | $-1 \le x < -\frac{1}{2}$ | $-1$ |
| $0 \le 2x+1 < 1$ | $-\frac{1}{2} \le x < 0$ | $0$ |
| $1 \le 2x+1 < 2$ | $0 \le x < \frac{1}{2}$ | $1$ |
| $2 \le 2x+1 < 3$ | $\frac{1}{2} \le x < 1$ | $2$ |
| $3 \le 2x+1 < 4$ | $1 \le x < \frac{3}{2}$ | $3$ |
| $\vdots$ | $\vdots$ | $\vdots$ |



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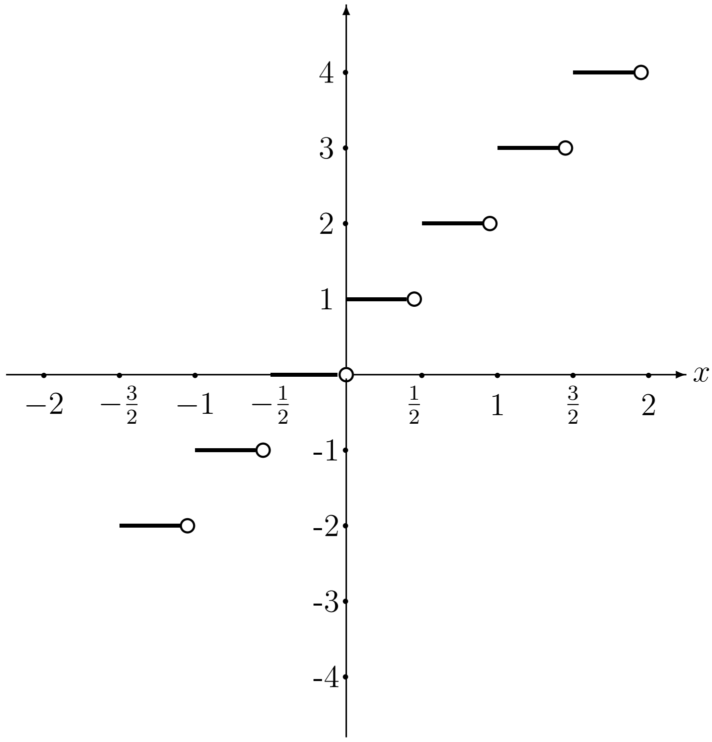
| $y = [[2x]] + 1$ | | |
|---|---|---|
| $2x$ | $x$ | $y = [[2x]] + 1$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $-3 \le 2x < -2$ | $-\frac{3}{2} \le x < -1$ | $-3 + 1 = -2$ |
| $-2 \le 2x < -1$ | $-1 \le x < -\frac{1}{2}$ | $-2 + 1 = -1$ |
| $-1 \le 2x < 0$ | $-\frac{1}{2} \le x < 0$ | $-1 + 1 = 0$ |
| $0 \le 2x < 1$ | $0 \le x < \frac{1}{2}$ | $0 + 1 = 1$ |
| $1 \le 2x < 2$ | $\frac{1}{2} \le x < 1$ | $1 + 1 = 2$ |
| $2 \le 2x < 3$ | $1 \le x < \frac{3}{2}$ | $2 + 1 = 3$ |
| $3 \le 2x < 4$ | $\frac{3}{2} \le x < 2$ | $3 + 1 = 4$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Note: This graph can be obtained by translating the graph of y=[[2x]] one unit upward. | | |

$y = [[2x]] + 1$




**Notes**: 1) The domain of all the above functions = $\Re$ while the range = the set of integers.
2) $[[x]] = x$ if and only if $x$ is an integer.
3) If n is an integer, then $y = [[mx + n]]$ and $y = [[mx]] + n$ have the same graph.
4) If E is an expression and n is an integer, then $[[E]] = n \implies n \le E < n + 1$.

**Problems**: 1) Solve the equations: a) $[[2x + 1]] + 1 = \frac{1}{2}$ , b) $-2[[2 - 3x]] - 5 = 7$

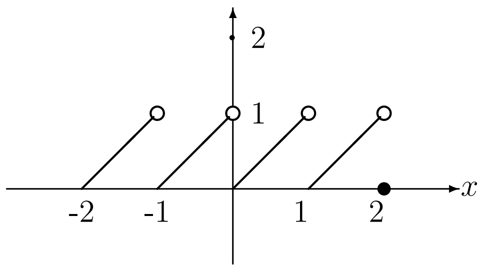
Solution: a) $[[2x + 1]] + 1 = \frac{1}{2} \implies [[2x + 1]] = \frac{1}{2} - 1 = -\frac{1}{2}$ is impossible, because $[[2x + 1]]$ must be an integer and $-\frac{1}{2}$ is not. Therefore S.S. = $\emptyset$

b) $-2[[2 - 3x]] - 5 = 7 \implies -2[[2 - 3x]] = 12 \implies [[2 - 3x]] = -6$
$\implies -6 \le 2 - 3x < -5 \implies -8 \le -3x < -7 \implies \frac{8}{3} \ge x > \frac{7}{3} \implies$ S.S. = $(\frac{7}{3}.\frac{8}{3}]$

2) Graph $y = x - [[x]]$ , $-2 \le x \le 2$. Write the domain and range.

Solution: $y = \begin{cases} x + 2 & \text{if } -2 \le x < -1 \\ x + 1 & \text{if } -1 \le x < 0 \\ x & \text{if } 0 \le x < 1 \\ x - 1 & \text{if } 1 \le x < 2 \\ 0 & \text{if } x = 2 \end{cases}$

$y = x - [[x]]$




Domain = [-2,2] , Range = [0,1)

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