Math 001 —— Common Mistakes

Important : Dear student,
below you will find a list of the most common mistakes that many stusents frequently do. I beieve that doing such mistakes like these is the main reason that some stusents are not be able to finish a problem or to get the right answer. What I want from each one of you is :
(i) read the common mistake and convince your self that this is a real mistake. If you are still not convinced, please check with me.
(ii) If you are convinced, read the correction of that mistake and compare the two.
(iii) Try your best to avoid these common mistakes. If you don't, then your grade in this course and may be in the next math courses will not be as good as you like.

1. $-(a-b) \neq -a-b$ but $-(a-b) = -a+b$.
2. $-\frac{x-y}{z} \neq \frac{-x-y}{z}$ but $-\frac{x-y}{z} = \frac{y-x}{z}$.
3. $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$ but $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$.
4. $\frac{a-b}{a+b} \neq -1$ but $\frac{a-b}{b-a} = -1$ and $\frac{-a-b}{a+b} = -1$.
5. π is not a **rational** number , this is because $\pi \neq 3.14$ or $\frac{22}{7}$. In fact $\pi \approx 3.14$ or $\frac{22}{7}$ and π is an **irrational** number .
6. For $a \neq 0$, $-a^0 \neq 1$ but $-a^0 = -1$ and $(-a)^0 = 1$.
7. $-(-a)^2 \neq a^2$ but $-(-a)^2 = -a^2$ and $-(-a)^3 = a^3$.
8. $ab^n \neq (ab)^n$, for example $2.3^2 \neq 36$ but $2.3^2 = 2.9 = 18$.
9. $ax^{-1} \neq \frac{1}{ax}$ but $ax^{-1} = \frac{a}{x}$ and $(ax)^{-1} = \frac{1}{ax}$.
10. $x^{-1} + y^{-1} \neq \frac{1}{x+y}$ but $(x+y)^{-1} = \frac{1}{x+y}$ and $x^{-1} + y^{-1} = \frac{y+x}{xy}$
11. $(a^x)^2 \neq a^{x^2}$ but $(a^x)^2 = a^{2x}$. In general $(a^x)^n = a^{nx}$.
12. $a^x.a^x \neq (a^2)^{2x}$ but $a^x.a^x = (a^2)^x = a^{2x}$, for example $3^x.3^x \neq 9^{2X}$ but $3^x.3^x = 9^x = 3^{2x}$
13. $a^{m^n} \neq (a^m)^n = a^{mn}$, for example $3^{2^4} = 3^{16}$ while $(3^2)^4 = 3^8$.
14. For $n \neq 1$, $(x+y)^n \neq x^n + y^n$, for example :
    (a) $(x+y)^2 \neq x^2 + y^2$ but $(x+y)^2 = x^2 + 2xy + y^2$.
    (b) $(x+y)^3 \neq x^3 + y^3$ but $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.
    (c) $(x^{-1} + y^{-1})^{-1} \neq x + y$ but $(x^{-1} + y^{-1})^{-1} = \frac{1}{x^{-1} + y^{-1}} \cdot \frac{xy}{xy} = \frac{xy}{y+x}$.
15. In general : $|x| \neq x$, because x is not always positive , for example $|-3| = |x| \neq x = -3$ but $|-3| = -x = -(-3) = 3$.
16. $-|-x| \neq |x|$ but $|-x| = |x|$ and $-|-x| = -|x|$.
17. In general : $|x+y| \neq |x| + |y|$ but $|x+y| \leq |x| + |y|$, for example $1 = |4 + (-3)| \leq |4| + |-3| = 7$.

18. $|x-y| \neq |x+y|$ but $|x-y| = |y-x|$ and $|-x-y| = |x+y|$. Therefore $\left|\frac{x-y}{y-x}\right| = 1$ and $\left|\frac{-x-y}{x+y}\right| = 1$ but $\left|\frac{x-y}{x+y}\right| \neq 1$.

19. $-\sqrt{-a} \neq \sqrt{a}$ while $-\sqrt[3]{-a} = \sqrt[3]{a}$.

20. $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$, e . g , $5 = \sqrt{9+16} \neq \sqrt{9} + \sqrt{16} = 7$.

21. $\sqrt[3]{x} \,.\, \sqrt[3]{x} \neq x$, $\sqrt[4]{x} \,.\, \sqrt[4]{x} \neq x$ but $(\sqrt[3]{x})^3 = x$ and $(\sqrt[n]{x})^n = x$ (whenever $\sqrt[n]{x}$ exists) .

22. $\frac{1}{\sqrt[3]{x}} \neq \frac{\sqrt[3]{x}}{x}$ but $\frac{1}{\sqrt[3]{x}} = \frac{\sqrt[3]{x^2}}{x}$. Similarly $\frac{1}{\sqrt[n]{x}} = \frac{\sqrt[n]{x^{(n-1)}}}{x}$.

23. In general : $\sqrt{x^2} \neq x$ but $\sqrt{x^2} = |x|$ and $\sqrt{(x-y)^2} = |x-y|$. In fact :

(a) $\sqrt[n]{x^n} = |x|$ if n is an **even** positive integer , e . g . , $\sqrt[4]{x^4} = |x|$.

(b) $\sqrt[n]{x^n} = x$ if n is an **odd** positive integer , e . g . , $\sqrt[5]{x^5} = x$.

24. $\sqrt[m]{x} \,.\, \sqrt[n]{x} \neq \sqrt[mn]{x}$ but $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$ while $\sqrt[m]{x} \,.\, \sqrt[n]{x} = x^{\frac{1}{m}} \,.\, x^{\frac{1}{n}} = x^{\frac{n+m}{mn}} = \sqrt[mn]{x^{n+m}}$.

25. $\sqrt[n]{x^n + y^n} \neq x + y$ and $(\sqrt[n]{x} + \sqrt[n]{y})^n \neq x + y$. But :

(a) $\sqrt[n]{(x+y)^n} = |x+y|$ if n is an **even** positive integer .

(b) $\sqrt[n]{(x+y)^n} = x + y$ if n is an **odd** positive integer .

(c) $(\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{xy} + y$ and so on .

26. $x^2 - y^2 \neq (x-y)^2$ and $x^2 + y^2 \neq (x+y)^2$ but :

(a) $(x+y)(x-y) = x^2 - y^2$.

(b) $(x-y)(x-y) = (x-y)^2 = x^2 - 2xy + y^2$.

(c) $(x+y)(x+y) = (x+y)^2 = x^2 + 2xy + y^2$.

27. $(\sqrt[3]{x} - \sqrt[3]{y}) \,.\, (\sqrt[3]{x} + \sqrt[3]{y}) \neq x - y$. In Fact :

(a) $(\sqrt{x} - \sqrt{y}) \,.\, (\sqrt{x} + \sqrt{y}) = x - y$.

(b) $(\sqrt[3]{x} - \sqrt[3]{y}) \,.\, (\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}) = x - y$.

(c) $(\sqrt[3]{x} + \sqrt[3]{y}) \,.\, (\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2}) = x + y$.

28. $\frac{1}{\sqrt[3]{x} - \sqrt[3]{y}} \neq \frac{\sqrt[3]{x} + \sqrt[3]{y}}{x-y}$ but $\frac{1}{\sqrt[3]{x} - \sqrt[3]{y}} = \frac{\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}}{x-y}$.

29. $x^3 - y^3 \neq (x-y)^3$ and $x^3 + y^3 \neq (x+y)^3$ but :

(a) $(x-y)^3 = (x-y)(x-y)^2 = (x-y)(x^2 - 2xy + y^2) = x^3 - 3x^2y + 3xy^2 - y^3$ while

(b) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.

(c) $(x+y)^3 = (x+y)(x+y)^2 = (x+y)(x^2 + 2xy + y^2) = x^3 + 3x^2y + 3xy^2 + y^3$ while

(d) $x^3+y^3=(x+y)(x^2-xy+y^2)$.

30. $\frac{x^3-y^3}{x^2-y^2} \neq x-y$ but $\frac{(x-y)^3}{(x-y)^2}=x-y$ and $\frac{x^3-y^3}{x^2-y^2}=\frac{(x-y).(x^2+xy+y^2)}{(x-y).(x+y)}=\frac{x^2+xy+y^2}{x+y}$

31. $\sqrt[3]{-1} \neq i$ but $\sqrt[3]{-1}=-1$ while $\sqrt{-1}=i$.

32. $\sqrt{-1}\ .\ \sqrt{-1} \neq \sqrt{(-1)(-1)}=\sqrt{1}=1$ but $\sqrt{-1}\ .\ \sqrt{-1}=i^2=-1$, similarly $\sqrt{-4}\ .\ \sqrt{-9}=6.i^2=-6$.

33. $\overline{2+\sqrt{3}} \neq 2-\sqrt{3}$ but $\bar{z}=z$ for any real number z , so $\overline{2+\sqrt{3}}=2+\sqrt{3}$.

34. $xy=1 \not\Longrightarrow x=1$ or $y=1$ but only $xy=0 \Longrightarrow x=0$ or $y=0$.

35. $(x-1)(x+3)=(x-1)(x+4) \not\Longrightarrow x+3=x+4$ but $\Longrightarrow (x-1)(x+3)-(x-1)(x+4)$ $=0 \Longrightarrow (x-1)(x+3-x-4)=-(x-1)=0 \Longrightarrow x=1$.

36. $a \geq b \not\Longrightarrow ax \geq bx$, this is because the **sign** of x is not known .

37. $ax > bx \not\Longrightarrow a > b$, for the same reason .

38. $\frac{x}{y} < \frac{z}{w} \not\Longrightarrow xw < yz$, because the sign of y and z is not known .

39. $\frac{1}{x} > \frac{1}{a} \not\Longrightarrow x > a$. In fact the solution is :

(a) For $a > 0$, we have $0 < x < a$, e . g . , $\frac{1}{x} > \frac{1}{2} \Longrightarrow 0 < x < 2$.

(b) For $a < 0$, we have $x < a$ or $x > 0$, e . g . , $\frac{1}{x} > \frac{-1}{3} \Longrightarrow x < -3$ or $x > 0$.

40. $\frac{1}{a} < \frac{1}{x} < \frac{1}{b} \not\Longrightarrow a < x < b$. In fact the solution of the inequality $\frac{1}{a} < \frac{1}{x} < \frac{1}{b}$ is :

(a) $b < x < a$ if a and b have the **same sign** , e . g . ,
$\frac{-1}{2} < \frac{1}{x} < \frac{-1}{3} \Longrightarrow -3 < x < -2$.

(b) $x < a$ or $x > b$ if $a < 0$ and $b > 0$, e . g . ,
$\frac{-1}{4} < \frac{1}{x} < \frac{1}{5} \Longrightarrow x < -4$ or $x > 5$.

41. The statements $x^2 < 9 \Longrightarrow x < \pm 3$ and $x^2 > 16 \Longrightarrow x > \pm 4$ are **meaningless** . The correction is :

(a) $x^2 < 9 \Longleftrightarrow |x| < 3 \Longrightarrow -3 < x < 3$ and

(b) $x^2 > 16 \Longleftrightarrow |x| > 4 \Longrightarrow x < -4$ or $x > 4$.

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Exercise : Now you are able to answer true (T) or false (F) for each of the following :

1. $\frac{4}{\pi}$ is a rational number .
2. $\frac{\sqrt{8}}{\sqrt{50}}$ is an irrational number .
3. $\{0, -1\}$ is closed under addition .
4. The multiplication inverse of $\frac{-\sqrt{2}}{2}$ is $-\sqrt{2}$.
5. $\frac{x-4}{x+4} = -1$.
6. $\frac{x^2-4}{x-2} = x-2$.
7. $\frac{x^3-8}{x-2} = x^2+4x+4$.
8. If $x = 2$, then $(x-2)^0 = 1$.
9. $-3^0 - (-4)^0 = -2$.
10. $(\frac{-8}{27})^{\frac{-2}{3}} = -\frac{9}{4}$.
11. $x.(x^{\frac{-1}{2}} - x^{\frac{1}{2}}).(x^{\frac{-1}{2}} + x^{\frac{1}{2}}) = 1 - x^2$.
12. $-||-x|| = -|x|$.
13. $|6-x| = x-6$ if $x < 6$.
14. $-|-x^2-3| = x^2+3$.
15. $\frac{1}{x^{-1}+y^{-1}} = x+y$.
16. $|x-9| = |9-x|$ and $|-y-6| = |y+6|$
17. $\frac{|x^4-16|}{|x^2-4|} = x^2+4$.
18. $-\sqrt{1-x} = \sqrt{x-1}$.
19. $\sqrt{x^2+16} = |x|+4$.
20. $\sqrt{(x+1)^2} = x+1$.
21. $\sqrt{(\pi-4)^2(\sqrt{5}-2)^2} = (4-\pi)(\sqrt{5}-2)$.
22. $\sqrt[3]{x^3+3x^2+3x+1} = x+1$.
23. $\sqrt{x^2-6x+9} = |x-3|$.
24. $(\sqrt{5}-\sqrt{3})^2 = 2$.

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25. $(\sqrt[3]{2}-1)^3 = 1 - 3.\sqrt[3]{4} + 3.\sqrt[3]{2}$.

26. $(\sqrt[3]{4}-\sqrt[3]{5}).(\sqrt[3]{16}+\sqrt[3]{20}+\sqrt[3]{25}) = -1$.

27. $\sqrt{x}.\sqrt[3]{x}.\sqrt[6]{x} = x$ for $x > 0$.

28. $\sqrt{\left(x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right)^2 - 4} = |x^{\frac{1}{2}} - x^{\frac{-1}{2}}|$ for $x > 0$.

29. $\dfrac{1}{\sqrt[4]{2}} = \dfrac{\sqrt[4]{2}}{2}$.

30. $\dfrac{\sqrt[3]{2}}{\sqrt[3]{2}-1} = 2 + \sqrt[3]{4} + \sqrt[3]{2}$.

31. $(\sqrt[3]{-1} + \sqrt{-1})^2 = -2i$.

32. $(x+1)(x-2) = 1 \implies x = 0$ or $x = 3$.

33. $(x+1)^2 = 4 \implies x = 1$.

34. $-(x+4) > -1 \implies x > -3$.

35. $\dfrac{x}{y} < 2 \implies x < 2y$.

36. $\dfrac{1}{|x|} < 1 \implies |x| > 1$.

37. $\dfrac{1}{x} > 3 \implies x < \dfrac{1}{3}$.

38. $\dfrac{1}{x^2} < \dfrac{1}{9} \implies x < -3$ or $x > 3$.

39. $1 \leq x^2 \leq 4 \implies -2 \leq x \leq -1$ or $1 \leq x \leq 2$.

40. $-1 < \dfrac{1}{x} < \dfrac{1}{3} \implies -1 \leq x \leq 3$.

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