

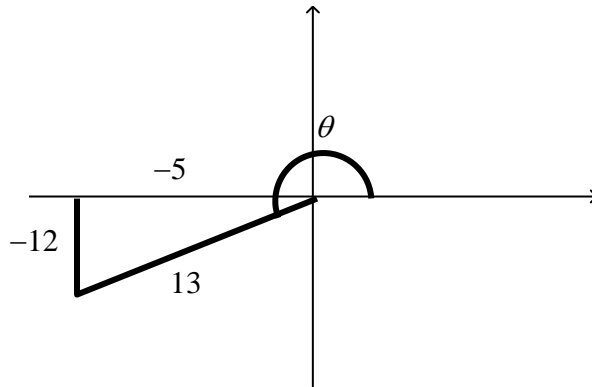
King Fahd University of Petroleum and Minerals
Perp-Year Math Program
Math 002 - Term 062
Recitation Hour (5.3 & 5.4)

Question1:

If the terminal side of an angle θ passes through the point $(-5, -12)$, then

find $\frac{\sec \theta - \tan \theta}{\cos \theta + \sin \theta}$.

Solution



$$\sec \theta = -\frac{13}{5}$$

$$\tan \theta = \frac{12}{5}$$

$$\cos \theta = -\frac{5}{13}$$

$$\sin \theta = -\frac{12}{13}$$

$$\frac{\sec \theta - \tan \theta}{\cos \theta + \sin \theta} = \frac{\frac{-13}{5} - \frac{12}{5}}{\frac{-5}{13} + \frac{-12}{13}} = \frac{65}{17}$$

Question2:

a) Find the measure of the reference angle θ' for the given angle θ

1. -570°

Solution

$$\theta = -570^\circ + 2(360^\circ) = 150^\circ \Rightarrow \theta' = 180^\circ - 150^\circ = 30^\circ$$

2. 30 radians

Solution

$$\theta = 30 = 4(2\pi) + 4.88 \Rightarrow \theta \in IV \ Q$$

$$\theta' = 2\pi - (30 - 8\pi) = 10\pi - 30$$

b) Find the exact value of $\cos 44^\circ + \cos 136^\circ + \sin(-510^\circ)$.

Solution

$$\begin{aligned}\cos 44^\circ + \cos 136^\circ + \sin(-510^\circ) &= \cos 44^\circ - \cos 44^\circ - \sin 510^\circ \\ &= -\sin 150^\circ \\ &= -\sin 30^\circ \\ &= -\frac{1}{2}\end{aligned}$$

c) Find the exact value of $\cos \frac{31\pi}{4} - \cos \frac{23\pi}{3} - \csc(-570^\circ)$.

Solution

$$\begin{aligned}\cos \frac{31\pi}{4} - \cos \frac{23\pi}{3} - \csc(-570^\circ) &= \cos\left(7\pi + \frac{3\pi}{4}\right) - \cos\left(5\pi + \frac{3\pi}{4}\right) + \csc 570^\circ \\ &= \cos\left(\pi + \frac{3\pi}{4}\right) - \cos\left(\pi + \frac{3\pi}{4}\right) + \csc 210^\circ \\ &= -\csc 30^\circ \\ &= -\frac{1}{2}\end{aligned}$$

Question3:

a) If W is the wrapping function, then find $W(-\frac{19\pi}{3})$.

Solution

$$\begin{aligned}W(-\frac{19\pi}{6}) &= \left(\cos(-\frac{19\pi}{6}), \sin(-\frac{19\pi}{6}) \right) \\&= \left(\cos(\frac{19\pi}{6}), -\sin(\frac{19\pi}{6}) \right) \\&= \left(\cos(3\pi + \frac{\pi}{6}), -\sin(3\pi + \frac{\pi}{6}) \right) \\&= \left(\cos(\pi + \frac{\pi}{6}), -\sin(\pi + \frac{\pi}{6}) \right) \\&= \left(-\cos \frac{\pi}{6}, \sin \frac{\pi}{6} \right) \\&= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2} \right)\end{aligned}$$

b) If $\cos 170^\circ = k$, then find the value of $\cos 350^\circ + 2 \csc 190^\circ$, in terms of k .

Solution

- $\cos 170^\circ = k \Rightarrow -\cos 10^\circ = k \Rightarrow \cos 10^\circ = -k$
- $\cos 350^\circ = \cos 10^\circ = -k$
- $\csc 190^\circ = -\csc 10^\circ = -\frac{1}{\sin 10^\circ} = -\frac{1}{\sqrt{1-\cos^2 10^\circ}} = -\frac{1}{\sqrt{1-k^2}}$

$$\therefore \cos 350^\circ + 2 \csc 190^\circ = -k - \frac{1}{\sqrt{1-k^2}}$$

Question4:

a) Write $\csc t$ in terms of $\tan t$, $\pi < t < \frac{3\pi}{2}$.

Solution

$$\csc t = \frac{1}{\sin t} \cdot \frac{\cos t}{\cos t} = \frac{\cos t}{\sin t} \cdot \frac{1}{\cos t} = \frac{1}{\tan t} \cdot \sec t$$

$$\text{but } 1 + \tan^2 t = \sec^2 t \Rightarrow \sec t = \pm \sqrt{1 + \tan^2 t}.$$

$$\text{Since } \pi < t < \frac{3\pi}{2} \Rightarrow \sec t = -\sqrt{1 + \tan^2 t}.$$

$$\therefore \csc t = \frac{-\sqrt{1 + \tan^2 t}}{\tan t}$$

b) Determine whether the function $f(x) = x \tan x$ is even, odd, or neither.

Solution

$$f(-x) = (-x) \tan(-x) = x \tan x = f(x) \Rightarrow f(x) \text{ is an even function}$$