

King Fahd University of Petroleum and Minerals
Prep-Year Math Program
Math 002 - Term 062
Recitation Hour (4.2 & 4.3)

Question1:

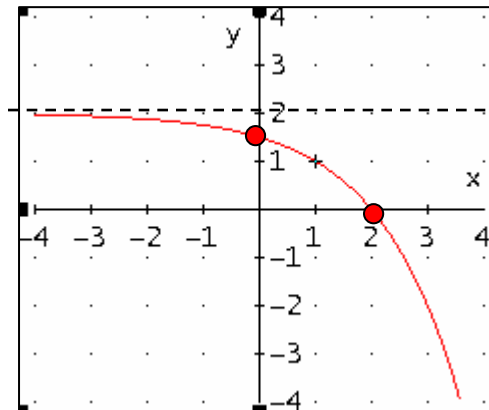
For the function

$$f(x) = -\left(\frac{1}{2}\right)^{-x+1} - 2$$

1) sketch the graph of $f(x)$

Solution

- H.A.: $y = -2$
- $x = 0 \Rightarrow y = \frac{3}{2}$
- $y = 0 \Rightarrow -\left(\frac{1}{2}\right)^{-x+1} + 2 = 0 \Rightarrow 2^{x-1} = 2 \Rightarrow x-1=1 \Rightarrow x = 2$



2) find the x – intercept and the y – intercept

Solution

- $y = 0 \Rightarrow -\left(\frac{1}{2}\right)^{-x+1} + 2 = 0 \Rightarrow 2^{x-1} = 2 \Rightarrow x-1=1 \Rightarrow x = 2$
- \therefore The x – intercept: $(2,0)$

$$\bullet \quad x = 0 \Rightarrow y = \frac{3}{2}$$

\therefore The y -intercept: $(0, \frac{3}{2})$

3) find the range

Solution

$$R = (-\infty, 2)$$

4) find the asymptote(s)

Solution

$$\text{H.A.: } y = -2$$

5) find the inverse function $f^{-1}(x)$

Solution

$$x = -\left(\frac{1}{2}\right)^{-y+1} + 2$$

$$\Rightarrow -\left(\frac{1}{2}\right)^{-y+1} = x - 2$$

$$\Rightarrow \left(\frac{1}{2}\right)^{-y+1} = -x + 2$$

$$\Rightarrow -y + 1 = \log_{1/2}(-x + 2)$$

$$\Rightarrow y = 1 - \log_{1/2}(-x + 2)$$

$$\therefore f^{-1}(x) = 1 - \log_{1/2}(-x + 2)$$

Question2:

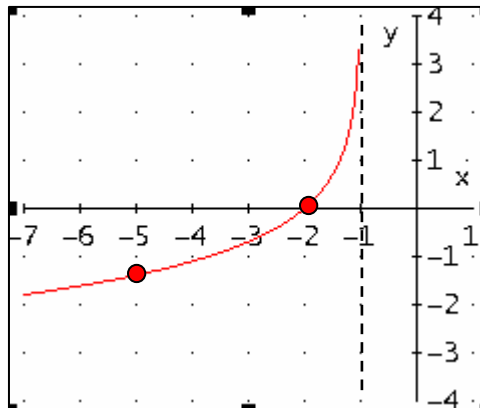
For functions

$$f(x) = -\log_2(-x-3)$$

1) sketch the graph of $f(x)$

Solution

- V.A.: $-x-3=0 \Rightarrow x=-3$
- $x=-4 \Rightarrow y=0$
- $x=-5 \Rightarrow y=-\log_2 2 = -1$



2) find , if any, the x – intercept and the y – intercept

Solution

- $y=0 \Rightarrow -\log(-x-3) = 0 \Rightarrow \log(-x-3) = 0 \Rightarrow -x-3 = 10^0 = 1$
 $\Rightarrow x = -4$

\therefore The x – intercept: $(-4, 0)$

- $x=0 \Rightarrow y = -\log(-3)!!!!$

\therefore No y – intercept

3) find the domain

Solution

$$-x - 1 > 0 \Rightarrow -x > 1 \Rightarrow x < -1$$

$$\therefore D = (-\infty, -1)$$

4) find the asymptote(s)

Solution

$$\text{V.A.: } x = -3$$

5) find the inverse function $f^{-1}(x)$

Solution

$$x = -\log(-y - 3)$$

$$\Rightarrow -x = \log(-y - 3)$$

$$\Rightarrow -y - 3 = 10^{-x}$$

$$\Rightarrow y = -10^{-x} - 3$$

$$\therefore f^{-1}(x) = -10^{-x} - 3$$

Question3:

If the graph of the exponential function $f(x) = b^x$ passes through the point $(-3, \frac{1}{64})$, then find $f(2)$.

Solution

The point $(-3, \frac{1}{64})$ on the graph

$$\Rightarrow \log_b \frac{1}{64} = -3 \Rightarrow \frac{1}{64} = b^{-3} \Rightarrow 4^{-3} = b^{-3} \Rightarrow b = 4.$$

$$\therefore f(x) = \log_4 x.$$

$$\therefore f(2) = \log_4 2 = \frac{1}{2}$$

Question4:

If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $N(t) = 100 \cdot 2^{t/3}$.

When will the population reach 6400?

Solution

$$128000 = 100 \cdot 2^{t/3}$$

$$\Rightarrow 128 = 2^{t/3}$$

$$\Rightarrow 2^7 = 2^{t/3}$$

$$\Rightarrow 7 = \frac{t}{3}$$

$$\Rightarrow t = 21 \text{ hours}$$