King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 062

Recitation Hour Problems (8.1 & 8.2)

Question1

Find the vertex, focus, and directrix of the parabola given by the equation: $6y - 3x^2 - 12x + 4 = 0$.

Question2

Find the equation in standard form of the parabola that has vertex $(-4,\ 1)$, has its axis of symmetry parallel to the y-axis , and passes through the point $(-2,\ 2)$.

Question3

Find the center, vertices, foci, and eccentricity of the ellipse given by the equation $9x^2 + y^2 + 18x - 6y + 9 = 0$.

Question4

Let P(3, 1) be a point on an ellipse that has foci at $F_1(-1,4)$ and $F_2(-1,-2)$, then find the length L of the major axis and the eccentricity e of the ellipse.

- Recitation 8.1 and 8.2

Q1.

$-3x^2-12x = -6y-4$

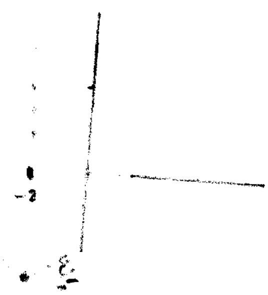
$\Rightarrow 3x^2+12x = 6y+4$, divide both sides by 3

$\Rightarrow x^2+4x = 2y+\frac{4}{3}$ complete the square for x,

$\Rightarrow x^2+4x+4 = 2y+\frac{4}{3}+4$

$\Rightarrow (x+2)^2 = 2y+\frac{16}{3}$

$(x+2)^2 = 2(y+\frac{8}{3})$



$4p=2 \Rightarrow p=\frac{1}{2}$.

vertex: $(-2, \frac{-8}{3})$

Focus: $(-2, \frac{-8}{3}+\frac{1}{2}) = (-2, \frac{-13}{6})$

Directrix: $y = \frac{-8}{3}-\frac{1}{2} = \frac{-19}{6}$

Q2. Check your notes.

Q3. $(9x^2+18x)+(y^2-6y) = -9$

$9(x^2+2x+1) + y^2-6y+9 = -9+9+9$

$9(x+1)^2 + (y-3)^2 = 9$

$(x+1)^2 + \frac{(y-3)^2}{9} = 1$

$\Rightarrow$ the major axis is parallel to the y-axis

$\left.\begin{array}{l} a^2=9 \Rightarrow a=3 \\ b^2=1 \Rightarrow b=1 \end{array}\right\}$ $c^2=a^2-b^2 = 9-1 = 8 \Rightarrow c=\sqrt{8}$

center: $(h,k) = (-1,3)$

vertices: $(h, k \mp a) = (-1, 3 \mp 3)$ $\to (-1,0)$, $(-1,6)$

Foci : $(h, k \mp c) = (-1, 3 \mp \sqrt{8})$

$e = \frac{c}{a} = \frac{\sqrt{8}}{3}$.

Q4.

- Method 1

Using the definition of the ellipse.

$d(P,F_1) + d(P,F_2) = 2a$

$\sqrt{(3+1)^2 + (1-4)^2} + \sqrt{(3+1)^2 + (1+2)^2} = 2a$

$\sqrt{16+9} + \sqrt{16+9} = 2a$

$5+5 = 2a$

$10 = 2a$

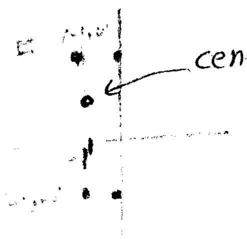
$\Rightarrow a = 5$.

$\therefore L = 2a = 10$

$e = \frac{c}{a}$. center: $\left(-1, \frac{4+(-2)}{2}\right) = \left(-1, \frac{2}{2}\right) = (-1,1)$

$c = 4-1 = 3$.

$\therefore e = \frac{3}{5}$.



center $\left(-1, \frac{4+(-2)}{2}\right) = (-1,1)$

$c = 4-1 = 3$

$c^2 = a^2 - b^2 \Rightarrow 9 = a^2 - b^2$

$\Rightarrow \boxed{a^2 = 9 + b^2}$

- Method 2:

Eq.: $\frac{(x+1)^2}{b^2} + \frac{(y-1)^2}{a^2} = 1$

$(3,1)$ on the ellipse

$\Rightarrow \frac{16}{b^2} + 0 = 1 \Rightarrow b^2 = 16 \Rightarrow a^2 = 9 + b^2 \Rightarrow a^2 = 9 + 16 = 25 \Rightarrow a = 5$

$\therefore L = 2a = 10$ and $e = \frac{c}{a} = \frac{3}{5}$.