

MATH 002 - T022 (EXAM I)

MASTER

Part I: (6-points) MULTIPLE CHOICE QUESTIONS (MCQ)**[Bubble the correct answer on the OMR sheet]**1. Which one of the following statements is TRUE for **any nonzero real numbers x and y** ?

(a) $\log\left(\frac{x}{y}\right)^2 = 2\log|x| - 2\log|y|$

(b) $\log\sqrt{x^2 + y^2} = \log|x| + \log|y|$

(c) $\log 10^{x+y} = |x + y|$

(d) $10^{\log|x| + \log|y|} = x + y$

On the Properties of the logarithmic and exponential functions.

2. The graph of the function $f(x) = \ln|x|$ is **increasing** on the interval:

(a) $(0, \infty)$

(b) $(-\infty, 0)$

(c) $(-\infty, 0) \cup (0, \infty)$

(d) $(-1, 0) \cup (0, 1)$

Problem #64 P.328

3. The length of an arc that subtends a central angle of 99° in a circle of radius 3 feet is equal to:

(a) $\frac{33\pi}{20}$ feet

(b) $\frac{60\pi}{11}$ feet

(c) $\frac{60}{11\pi}$ feet

(d) 297 feet

Similar to Example 5 P.396

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4. If $f(x) = \log_4 x$, then $f^{-1}\left(\frac{3}{2}\right)$ is equal to:

(a) 8

(b) 6

(c) $\frac{8}{3}$ (d) $2\sqrt[3]{2}$

similar to Example #1(a) p. 321

5. Given the angles $\alpha = 51^\circ 49'$ and $\beta = 38^\circ 11'$, then which one of the following statements is TRUE?

(a) α and β are complementary angles(b) α and β are supplementary angles(c) $\alpha + \beta$ is an acute angle(d) $\alpha + \beta$ is an obtuse angle

similar to Example #1 p. 390

& #5 p. 399

6. The exact value of $\sin \frac{\pi}{3} - \csc \frac{\pi}{3} \sin \frac{\pi}{6}$ is equal to:

(a) $\frac{\sqrt{3}}{6}$ (b) $\frac{7\sqrt{3}}{6}$ (c) $-\frac{\sqrt{3}}{6}$ (d) $-\frac{2\sqrt{3}}{3}$

similar to #38 p. 411

Part II: (6-POINTS) TRUE AND FALSE STATEMENTS**[Bubble the correct answer on the OMR sheet. A for TRUE and B for FALSE.]**7. If $\cot \theta < 0$ and $\sec \theta > 0$, then $\sin \theta > 0$.

A: TRUE

 B: FALSE*Similar to # (9 to 14) p. 421*8. The expression $e^{-1+\ln 2}$ is equal to $\frac{2}{e}$. A: TRUE

B: FALSE

9. If the terminal side of an angle θ contains the point $P(-2, \frac{1}{3})$, then $\sin \theta = \frac{1}{3}$.

A: TRUE

 B: FALSE*Similar to Example #1 p. 416
& #6 p. 421*10. The reference angle of the angle -30° is equal to 330° .

A: TRUE

 B: FALSE*Similar to Example #3 p. 419
and #6 p. 422*11. The product $(\log_5 3)(\log_3 5)(\log_5 25)$ is equal to 2. A: TRUE

B: FALSE

*similar to # 72 p. 391*12. The function $f(x) = x^3 - \tan x$ is an odd function. A: TRUE

B: FALSE

similar to #33 to 40 p. 432

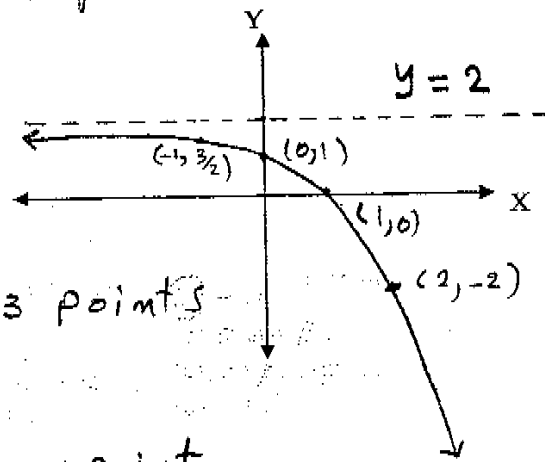
PART III: WRITTEN QUESTIONS

[Provide neat and complete solution to each question. Show necessary steps for full credit.]

1. (6-points) Sketch the graph of the function $f(x) = -2^x + 2$, and find its domain, range, x - intercept, y - intercept, and the asymptotes as indicated below:

a) The graph

The graph of $f(x) = -2^x + 2$ is the graph of $g(x) = 2^x$ reflected across the x-axis and shifted up vertically 2 units



b) The domain = $(-\infty, \infty)$
The range = $(-\infty, 2)$ } ... 1 point

c) The x - intercept: $f(x) = 0 \Rightarrow 2^x = 2 \Rightarrow x = 1 \Rightarrow$ x-intercept = $(1, 0)$
The y - intercept: $x = 0 \Rightarrow f(0) = -1 + 2 = 1 \Rightarrow$ y-intercept = $(0, 1)$
... 1 point

d) The Asymptotes: $y = 2$, No other asymptotes.
... 1 point

2. (3-points) Find the exact value of $\csc \frac{19\pi}{6} + \tan 144^\circ + \tan 36^\circ$. (Show your steps)

$\frac{19\pi}{6} = 3\pi + \frac{\pi}{6} \Rightarrow \frac{19\pi}{6}$ in a Quadrant III angle whose reference angle is $\frac{\pi}{6}$... 1 point

and 146° in a Quadrant II angle whose reference angle is $180^\circ - 146^\circ = 36^\circ$... 1 point

\Rightarrow The given expression = $-\csc \frac{\pi}{6} - \tan 36^\circ + \tan 36^\circ = -2$
Similar to Example 4 p.420 and #37 to 48 p.422 ... 1 point

3. (3-points) Write the following expression as a single natural logarithm:

$$-2 + \frac{1}{3} \ln \left(\frac{x^3}{y^6} \right) + 2 \ln(xye)$$

$$\text{The given expression} = \ln e^{-2} + \ln \left(\frac{x^3}{y^6} \right)^{\frac{1}{3}} + \ln (xye)^2$$

--- 1 point

$$= \ln \left[e^{-2} \cdot \frac{x}{y^2} \cdot x^2 y^2 e^2 \right]$$

--- 1 point

$$= \ln x^3$$

--- 1 point

Similar to #11 to 20 p. 339-340

4. (3-points) Let W be the wrapping function, and t be a real number with $\frac{3\pi}{2} < t < 2\pi$.

If $W(t) = P\left(\frac{\sqrt{5}}{3}, b\right)$, then find the value of b .

The point $P\left(\frac{\sqrt{5}}{3}, b\right)$ lies on the unit circle $x^2 + y^2 = 1$
and in Quadrant IV --- 1 point

$$\Rightarrow \frac{5}{9} + b^2 = 1 \Rightarrow b^2 = \frac{4}{9} \Rightarrow b = \pm \frac{2}{3} \text{ --- 1 point}$$

$$\Rightarrow b = -\frac{2}{3} \text{ only --- 1 point}$$

See the definition of W + #1 to 12 p. 431

5. (3-points) Express $\tan \theta - \frac{\sec^2 \theta}{\tan \theta}$ in terms of $\cot \theta$. (Show your steps).

$$\text{The expression} = \frac{\tan^2 \theta - \sec^2 \theta}{\tan \theta} \text{ --- 1 point}$$

$$= \frac{-1}{\tan \theta} \text{ --- 1 point}$$

$$= -\cot \theta \text{ --- 1 point}$$

similar to #49 to 62 p. 432

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6. (4-points) Solve: $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{4}$

$$\Rightarrow 4e^x - 4e^{-x} = e^x + e^{-x} \quad \dots 1 \text{ point}$$

$$\Rightarrow 3e^x = 5e^{-x} \quad \dots 1 \text{ point}$$

$$\Rightarrow e^{2x} = \frac{5}{3} \quad \dots 1 \text{ point}$$

$$\Rightarrow 2x = \ln \frac{5}{3} \Rightarrow x = \frac{1}{2} \ln \frac{5}{3} \quad \dots 1 \text{ point}$$

Similar to #39 to 46 p. 349

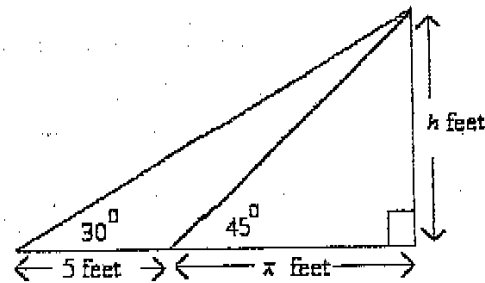
7. (3-points) Find the exact value of the height h of the right triangle shown in the figure:
(Show your steps)

From the figure \Rightarrow

$$x = h \quad \dots 1 \text{ point}$$

$$\text{and } \tan 30^\circ = \frac{h}{5+x}$$

$$= \frac{h}{5+h} = \frac{1}{\sqrt{3}} \quad \dots 1 \text{ point}$$



$$\Rightarrow \sqrt{3}h = 5+h \Rightarrow (\sqrt{3}-1)h = 5 \Rightarrow h = \frac{5}{\sqrt{3}-1} \quad \dots 1 \text{ point}$$

Similar to Example 6 p. 409

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8. (3-points) A wheel is rotating at 100 revolutions per minute. Find the exact angular speed of the wheel in radians per seconds. (show your steps).

$$100 \text{ rev/minute} = \frac{100 \text{ revolutions}}{1 \text{ minute}} \left(\frac{2\pi \text{ radians}}{1 \text{ rev}} \right) \left(\frac{1 \text{ minute}}{60 \text{ seconds}} \right)$$

--- 2 points

similar to
#69 p.400
and example 7 p.397

$$= \frac{200\pi}{60} \text{ radians/second}$$

$$= \frac{10\pi}{3} \text{ radians/second} \quad \text{--- 1 point}$$

9. (4-points) Solve the logarithmic equation:

$$\log(x+5) + \log(x-2) = \log(12x)$$

$$\Rightarrow \log[(x+5)(x-2)] = \log(12x) \quad \text{--- 1 point}$$

$$\Rightarrow (x+5)(x-2) = 12x$$

$$\Rightarrow x^2 + 3x - 10 = 12x$$

$$\Rightarrow x^2 - 9x - 10 = 0 \quad \text{--- 1 point}$$

$$\Rightarrow (x-10)(x+1) = 0$$

$$\Rightarrow x=10 \quad \text{or} \quad x=-1 \quad \text{--- 1 point}$$

check $\Rightarrow x=10$ is the only solution of
the given equation
--- 1 point

similar to #21 to 30 p.349