King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101-Term 043
Exam II
Duration: 75 Minutes

KEY

Name: ______________________	Serial #:__________
ID #:______________________	Section:__________

Question #	Mark	Total Mark
1		4
2		2
3		4
4		2
5		3
6		3
Total Marks		**18**

Question1 (1pt. each)

a) If $f(x) = x^6 - 2x^2 - 3$, then find $\lim_{x\to 2} \frac{f'(x) - f'(2)}{x-2}$ $= f''(2) = 476$

Sol.: $f'(x) = 6x^5 - 4x$

$f''(x) = 30x^4 - 4$

$f''(2) = 30(16) - 4 = 476$ (1)

b) If $y = \cot 2x$, then find y'' at $x = \frac{\pi}{8}$

Sol.: $y' = -\csc^2 2x \cdot 2$

$= -2\csc^2 2x$

$y'' = -4 \csc 2x \cdot -\csc 2x \cot 2x \cdot 2$

$y'' = 8 \csc^2 2x \cot 2x$

$y''\Big|_{x=\frac{\pi}{8}} = 8 \cdot (2)(1) = 16$ (1)

c) $\frac{d^{87}}{dx^{87}}[2\cos x]$

Sol.: $y = \cos x \Rightarrow y' = -\sin x$

$y'' = -\cos x$

$y''' = \sin x$

$y^{(4)} = \cos x$

$4 \overline{)87}$ = 21, remainder 3

$\therefore \frac{d^{87}}{dx^{87}}[2\cos x] = 2\frac{d^{87}}{dx^{87}}[\cos x] = 2\frac{d^3}{dx^3}[\cos x] = 2\sin x$ (1)

d) If $u\cos(v+u) = 1$ find $\frac{du}{dv}$

Sol.: $\frac{du}{dv}\cos(v+u) + u \cdot -\sin(v+u) \cdot \left[1 + \frac{du}{dv}\right] = 0$

$\frac{du}{dv}\cos(v+u) - u\sin(v+u) - u\sin(v+u)\frac{du}{dv} = 0$

$\frac{du}{dv}\left[\cos(v+u) - u\sin(v+u)\right] = u\sin(v+u)$

$\Rightarrow \frac{du}{dv} = \frac{u\sin(v+u)}{\cos(v+u) - u\sin(v+u)}$ ( )

Question2 (2pts)

Let $f(x)=\begin{cases} x^2 & if \ x<2 \\ ax+b & if \ x>2 \end{cases}$. Find the values of a and b that make f differentiable everywhere

Sol.: f is diff everywhere ⇒ f'(2) exist ⇒ (1) f cont at x = 2
(2) f'(2) exists

* f is cont at x = 2 ⇒ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$4 = 2a + b$ ----- (1)

* f'(2) exist ⇒ $f'_-(2) = f'_+(2)$ ⇒ $2(2) = a$ ⇒ $a = 4$ and $b = -4$ (1)

Question3 (2 pts each)

a) Find the equation of the tangent line to the curve $\frac{2y^2 - x^2}{x} = 1$ at the point $(2,2)$

Sol.: $2y^2 - x^2 = x$, differentiate both sides w.r.t. x

$4yy' - 2x = 1$ ----(1)

at (2,2) ⇒ $8y' - 4 = 1$ ... $6y' = 6$ ⇒ $y' = 1$ (0.5)

∴ Eq: $y - 2 = (x - 2)$

⇒ $y = x$ --- (0.5)

b) Find the equations of the line through the point $(2,0)$ that are tangent to the curve $y = 3 - x^2$

Sol.: slope $= y' = -2x$. Also the slope $= \frac{\Delta y}{\Delta x} = \frac{y-0}{x-2} = \frac{y}{x-2}$

∴ $-2x = \frac{y}{x-2}$ ⇒ $-2x = \frac{3-x^2}{x-2}$

⇒ $-2x^2 + 4x = 3 - x^2$ ⇒ $x^2 - 4x + 3 = 0$

⇒ $(x-1)(x-3) = 0$

⇒ $x = 1$, $x = 3$

⇒ $y = 2$, $y = -6$ (1)

(0.5) Eq1: at (1,2)

$y - 2 = -2(x - 1)$

$y - 2 = -2x + 2$

(0.5) Eq2: at (3,-6)

$y + 6 = -6(x - 3)$

$y + 6 = -6x + 18$

Question4 (2 pts)

Let g be a differentiable function with $g(1) = \frac{\pi}{8}$ and $g'(1) =$. If $f(x) = \frac{g(\tan x)}{x^2}$, then find $f'(\frac{\pi}{4})$.

Sol: $f'(x) = \frac{x^2 \cdot g'(\tan x) \cdot \sec^2 x - g(\tan x) \cdot 2x}{x^4}$ (1.5)

$f'(\frac{\pi}{4}) = \frac{(\frac{\pi}{4})^2 g'(1)(2) - g(1)\, 2 \cdot \frac{\pi}{4}}{(\frac{\pi}{4})^4}$

$= \frac{\frac{\pi^2}{16} - \frac{\pi}{8} \cdot \frac{\pi}{2}}{(\frac{\pi}{4})^4}$ (0.5)

$= 0$

Question5 (1.5 pts each)

a) Use the local linear approximation to estimate the value of $(0.97)^{5/3}$

Let $f(x) = x^{5/3}$, $x_0 = 1$

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$, $f'(x) = \frac{5}{3}x^{2/3}$

$\approx f(1) + f'(1)(x-1)$

$x^{5/3} \approx 1 + \frac{5}{3}(x-1)$

$x = 0.97 \Rightarrow (0.97)^{5/3} \approx 1 + \frac{5}{3}(0.97 - 1)$

$\approx 1 + \frac{5}{3}(-0.03)$

$\approx 1 - 0.05 = 0.95$

b) The side of a square is measured to be 10 ft, with possible error of ± 0.1 ft. Estimate the **percentage error** in the side and the **area**.

* Percentage error in the side $= \frac{dx}{x} * 100\% = \frac{\mp 0.1}{10} * 100\% = \mp 1\%$ (0.5)

* Percentage error in the area $= \frac{dA}{A} * 100\%$

$= \frac{2x\,dx}{x^2} * 100\%$

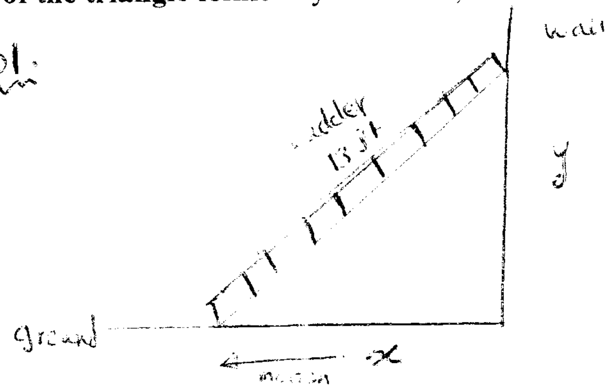
$= \frac{2\,dx}{x} * 100\%$ (1)

$= \frac{2(\mp 0.1)}{10} * 100\% = \mp 2\%$

Question6 (3 pts)

A 13- ladder is leaning agair:st a wall when its base starts t(lide away. By the time the base is 12 ft from the wal!. :he base is moving at the rate o ft/sec. At what rate is the **area of the triangle** formed by the ladder, wall and the groun :hanging?

Sol.



Given $\frac{dx}{dt} = 5$ ft/sec.

Find $\left.\frac{dA}{dt}\right|_{x=12} = ?$

$A = \frac{1}{2} x y \Rightarrow \frac{dA}{dt} = \frac{1}{2} \frac{dx}{dt} \cdot y \quad \frac{1}{2} x \frac{dy}{dt}$ .... (1)

× Find $\frac{dy}{dt}$.

$x^2 + y^2 = (13)^2$. Diffrenziate both sides with respect to t

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$ (2)

But $x^2 + y^2 = (13)^2$ (0.5)

$(12)^2 + y^2 = (13)^2$

$y^2 = 25 \Rightarrow y = 5$

$\Rightarrow \frac{dy}{dt} = -\frac{12}{5} \cdot 5 = -12$ ft/sec

Hence.

$\frac{dA}{dt} = \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$

$= \frac{25}{2} - \frac{144}{2}$

$= -\frac{119}{2}$ ft^2/sec .... (1.5)