King Fahd Univesity of Petroleum & Minerals
Department of Mathematical Sciences

KEY

Math 101-Term 043
Exam II
Duration: 75 Minutes

Name: ____________________ Serial #:__________

ID #:____________________ Section:__________

Question #	Mark	Total Mark
1		4
2		2
3		4
4		2
5		3
6		3
Total Marks		**18**

Question1 (1pt. each)

a) If $f(x)=x^6-2x^2+3$, then find $\lim_{x\to 2}\frac{f'(x)-f'(2)}{x-2}$ $= f''(2) = 476$

Sol.: $f'(x)=6x^5-4x$

$f''(x)=30x^4-4$

$f''(2)=30(16)-4=476$ 1

b) If $y=\cot 2x$, then find y'' at $x=\frac{\pi}{8}$

Sol.: $y'=-\csc^2 2x\cdot 2$

$=-2\csc^2 2x$

$y''=-4\csc 2x\cdot -\csc 2x\cot 2x\cdot 2$

$y''=8\csc^2 2x\cot 2x$

$y''\Big|_{x=\frac{\pi}{8}}=8\cdot(2)(1)=16$ 1

c) $\frac{d^{87}}{dx^{87}}[2\cos x]$

Sol.: $y=\cos x \Rightarrow y'=-\sin x$

$y''=-\cos x$

$y'''=\sin x$

$y^{(4)}=\cos x$

$$\begin{array}{r} 21 \\ 4\overline{)87} \\ \underline{8} \\ 07 \\ \underline{8} \\ 3 \end{array}$$

$\therefore \frac{d^{87}}{dx^{87}}[2\cos x]=2\frac{d^{87}}{dx^{87}}[\cos x]=2\frac{d^3\cos x}{dx^3}=2\sin x$ 1

d) If $u\cos(v+u)=1$ find $\frac{du}{dv}$

Sol.: $\frac{du}{dv}\cos(v+u)+u\cdot-\sin(v+u)\cdot\left[1+\frac{du}{dv}\right]=0$

$\frac{du}{dv}\cos(v+u)-u\sin(v+u)-u\sin(v+u)\frac{du}{dv}=0$

$\frac{du}{dv}\left[\cos(v+u)-u\sin(v+u)\right]=u\sin(v+u)$

$\Rightarrow \frac{du}{dv}=\frac{u\sin(v+u)}{\cos(v+u)-u\sin(v+u)}$ 1

Question2 (2pts)

Let $f(x)=\begin{cases} x^2 & \text{if } x\le 2 \\ ax+b & \text{if } x>2 \end{cases}$. Find the values of a and b that make f differentiable everywhere

Sol.: f is diff everywhere $\Rightarrow f'(2)$ exist $\Rightarrow$ (1) $f(x)$ is cont at $x=2$

(2) $f'(2)$ exists

* f is cont. at $x=1 \Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$4 = 2a+b$ ----- (1)

* $f'(2)$ exist $\Rightarrow f'_+(2) = f'_-(2) \Rightarrow 2(2) = a \Rightarrow \boxed{a=4}$ (1) and $\boxed{b=0}$

Question3 (2 pts each)

a) Find the equation of the tangent line to the curve $\frac{2y}{x}-\frac{x}{y}=1$ at the point $(2,2)$

Sol.: $2y^2-x^2=xy$, differentiate both sides w.r.to x

$4yy'-2x=y+xy'$ ----(1)

at $(2,2) \Rightarrow 8y'-4=2+2y'$

$\Rightarrow 6y'=6 \Rightarrow y'=1$ (0.5)

$\therefore$ Eq. $y-2=(x-2)$

$\Rightarrow \boxed{y=x}$ --- (0.5)

b) Find the equations of the line through the point $(2,0)$ that are tangent to the curve $y=3-x^2$

Sol.: slope $= y' = -2x$. Also the slope $= \frac{\Delta y}{\Delta x} = \frac{y-0}{x-2} = \frac{y}{x-2}$

$\therefore -2x = \frac{y}{x-2} \Rightarrow -2x = \frac{3-x^2}{x-2}$

$\Rightarrow -2x^2+4x=3-x^2 \Rightarrow x^2-4x+3=0$

$\Rightarrow (x-1)(x-3)=0$

$\Rightarrow x=1,\ x=3$

$\Rightarrow y=2,\ y=-6$ (1)

(0.5)

Eq1: at $(1,2)$

$y-2=-2(x-1)$

$y-2=-2x+2$

$y=-2x+4$

(0.5)

Eq.2: at $(3,-6)$

$y+6=-6(x-3)$

$y+6=-6x+18$

$y=-6x+12$

Question4 (2 pts)

Let g be a differentiable function with $g(1)=\frac{\pi}{8}$ and $g'(1)=\frac{1}{2}$. If $f(x)=\frac{g(\tan x)}{x^2}$, then find $f'(\frac{\pi}{4})$.

Sol.

$$f'(x) = \frac{x^2 \cdot g'(\tan x)\cdot \sec^2 x - g(\tan x)\cdot 2x}{x^4} \quad 1.5$$

$$f'(\tfrac{\pi}{4}) = \frac{(\frac{\pi}{4})^2 g'(1)(2) - g(1)\, 2\cdot\frac{\pi}{4}}{(\frac{\pi}{4})^4}$$

$$= \frac{\frac{\pi^2}{16} - \frac{\pi}{8}\cdot\frac{\pi}{2}}{(\frac{\pi}{4})^4} \quad 0.5$$

$$= 0$$

Question5 (1.5 pts each)

a) Use the local linear approximation to estimate the value of $(0.97)^{5/3}$

Let $f(x) = x^{5/3}$, $x_0 = 1$

$f(x) \simeq f(x_0) + f'(x_0)(x-x_0)$; $f'(x) = \frac{5}{3}x^{2/3}$

$\simeq f(1) + f'(1)(x-1)$

$x^{5/3} \simeq 1 + \frac{5}{3}(x-1)$

$x=0.97 \Rightarrow (0.97)^{5/3} \simeq 1 + \frac{5}{3}(0.97-1)$

$\simeq 1 + \cancel{\frac{5}{3}}\cdot \cancel{-0.03}$

$\simeq 1 - 0.05 = 0.95$

b) The side of a square is measured to be 10 ft, with possible error of ± 0.1 ft. Estimate the **percentage error** in the <u>side</u> and the <u>area</u>.

* Percentage error in the side $= \frac{dx}{x} * 100\% = \frac{\mp 0.1}{10} * 100\% = \mp 1\%$ 0.5

* percentage error in the area $= \frac{dA}{A} * 100\%$

$= \frac{2x\,dx}{x^2} * 100\%$

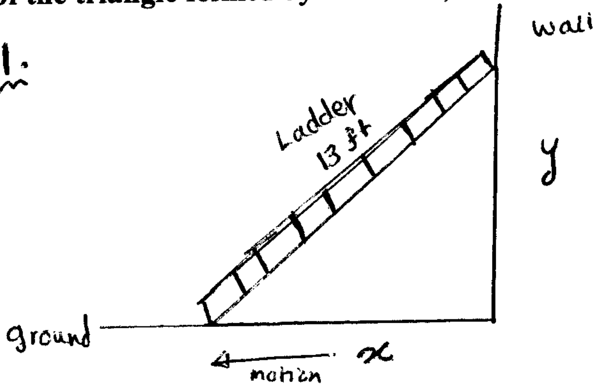
$= \frac{2dx}{x} * 100\%$

$= \frac{2(\mp 0.1)}{10} * 100\% = \mp 2\%$ 1

Question6 (3 pts)

A 13- ladder is leaning against a wall when its base starts to slide away. By the time the base is 12 ft from the wall, the base is moving at the rate of 5 ft/sec. At what rate is the **area of the triangle** formed by the ladder, wall and the ground changing?

Sol.



Given $\frac{dx}{dt} = 5$ ft/sec.

Find $\left.\frac{dA}{dt}\right|_{x=12} = ?$

$$A = \frac{1}{2}xy \Rightarrow \frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}\cdot y + \frac{1}{2}x\frac{dy}{dt} \quad \cdots \text{(1)}$$

* Find $\frac{dy}{dt}$.

$x^2 + y^2 = (13)^2$. Diffrentiate both sides with respect to t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\cdot\frac{dx}{dt} \quad (1)$$

. But $x^2 + y^2 = (13)^2$ (0.5)

$(12)^2 + y^2 = (13)^2$

$y^2 = 25 \Rightarrow y = 5$

$$\Rightarrow \frac{dy}{dt} = -\frac{12}{5}\cdot 5 = -12 \text{ ft/sec.}$$

Hence.

$$\frac{dA}{dt} = \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$$

$$= \frac{25}{2} - \frac{144}{2}$$

$$= -\frac{119}{2} \text{ ft}^2/\text{sec.} \quad \cdots \text{(0.5)}$$