6.

$$\begin{aligned} 3^{4x-7} &= \frac{1}{9} \\ 3^{4x-7} &= 3^{-2} \\ 4x-7 &= -2 \\ 4x &= 5 \\ x &= \frac{5}{4} \end{aligned}$$

14.

$$\begin{aligned} 10^{6-x} &= 550 \\ (6-x)\log 10 &= \log 550 \\ 6-x &= \log 550 \\ x &= 6-\log 550 \end{aligned}$$

20.

$$\begin{aligned} 5^{3x} &= 3^{x+4} \\ \log 5^{3x} &= \log 3^{x+4} \\ 3x\log 5 &= (x+4)\log 3 \\ 3x\log 5 &= x\log 3+4\log 3 \\ 3x\log 5-x\log 3 &= 4\log 3 \\ x(3\log 5-\log 3) &= 4\log 3 \\ x &= -\frac{4\log 3}{3\log 5-\log 3} \end{aligned}$$

30.

$$\begin{aligned} \log(4-x) &= \log(x+8)+\log(2x+13) \\ \log(4-x) &= \log[(x+8)(2x+13)] \\ 4-x &= (x+8)(2x+13) \\ 4-x &= 2x^2+29x+104 \\ 0 &= 2x^2+30x+100 \\ 0 &= 2(x^2+15x+50) \\ 0 &= 2(x+5)(x+10) \end{aligned}$$

$x=-5$ or $x=-10$ (No; not in domain.)

The solution is $x=-5$.

32.

$$\begin{aligned} \log x^3 &= (\log x)^2 \\ 3\log x &= (\log x)^2 \\ (\log x)^2-3\log x &= 0 \\ \log x(\log x-3) &= 0 \end{aligned}$$

$\log x = 0$ or $\log x - 3 = 0$

$x = 1$ $\quad$ $\log x = 3$

$x = 1000$

34.

$$\begin{aligned} \ln(\ln x) &= 2 \\ e^2 &= \ln x \\ e^{e^2} &= x \end{aligned}$$

44.

$$\frac{e^x - e^{-x}}{2} = 15$$

$$e^x(e^x - e^{-x}) = (30)(e^x)$$

$$e^{2x} - 1 = 30e^x$$

$$e^{2x} - 30e^x - 1 = 0$$

Let $u = e^x$.

$$u^2 - 30u - 1 = 0$$

$$u = \frac{30 \pm \sqrt{900 - 4(-1)}}{2}$$

$$u = \frac{30 \pm \sqrt{904}}{2} = \frac{30 \pm 2\sqrt{226}}{2}$$

$$u = 15 \pm \sqrt{226}$$

$$e^x = 15 \pm \sqrt{226}$$

$$x \ln e = \ln(15 \pm \sqrt{226})$$

$$x = \ln(15 + \sqrt{226})$$

45.

$$\frac{1}{e^x - e^{-x}} = 4$$

$$1 = 4(e^x - e^{-x})$$

$$1(e^x) = 4(e^x)(e^x - e^{-x})$$

$$e^x = 4(e^{2x} - 1)$$

$$e^x = 4e^{2x} - 4$$

$$0 = 4e^{2x} - e^x - 4$$

Let $u = e^x$.

$$0 = 4u^2 - u - 4$$

$$u = \frac{1 \pm \sqrt{1 - 4(4)(-4)}}{8}$$

$$u = \frac{1 \pm \sqrt{65}}{8}$$

$$e^x = \frac{1 + \sqrt{65}}{8}$$

$$x \ln e = \ln\left(\frac{1 + \sqrt{65}}{8}\right)$$

$$x = \ln(1 + \sqrt{65}) - \ln 8$$

78.

$$x = 500^{501} \qquad y = 506^{500}$$

$$\ln x = 501 \ln 500 \qquad \ln y = 500 \ln 506$$

A calculator shows that $\ln x > \ln y$. Thus $500^{501} > 506^{500}$.