

King Fahd University of Petroleum and Minerals
Faculty of Science – Math Prep Year program
Math 002 -042
Quiz #3 (5.4-5.6)

Name: _____ ID: _____ Sr#: _____ Sec.: _____

Question1

(4 points)

If W is the wrapping function, then find $W(-\frac{5\pi}{3})$.

Solution

$$W(-\frac{5\pi}{3}) = \left(\cos(-\frac{5\pi}{3}), \sin(-\frac{5\pi}{3}) \right) = (\cos(-300^\circ), \sin(-300^\circ)) = (\cos 60^\circ, \sin 60^\circ) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

Question2

(3 points)

Write $\tan t$ in terms of $\cos t$, $\pi < t < \frac{3\pi}{2}$.

Solution

$$\tan t = \frac{\sin t}{\cos t}, \text{ but } \sin^2 t + \cos^2 t = 1 \Rightarrow \sin t = \pm \sqrt{1 - \cos^2 t}, \text{ but } \pi < t < \frac{3\pi}{2} \Rightarrow \sin t = -\sqrt{1 - \cos^2 t}$$

$$\therefore \tan t = -\frac{\sqrt{1 - \cos^2 t}}{\cos t}$$

Question3

Let $f(x) = a \cos bx$ with period = 8 and $f(4) = 3$.

- Find the values of a and b .
- Find the range of $f(x)$.

(2 points)

(2 points)

Solution

$$a) P = \frac{2\pi}{|b|} = 8 \Rightarrow |b| = \frac{\pi}{4} \Rightarrow b = \pm \frac{\pi}{4}$$

$$\therefore f(x) = a \cos(\pm \frac{\pi}{4} x) = a \cos(\frac{\pi}{4} x)$$

$$f(4) = 3 \Rightarrow a \cos \pi = 3 \Rightarrow -a = 3 \Rightarrow a = -3$$

$$b) R = [-|a|, |a|] = [-3, 3]$$

Question4

For the function $f(x) = -2 \tan \frac{2x}{3}$,

- a) Find the amplitude. (1 point)
- b) Find $f(-\pi)$. (2 points)
- c) Find the equation of all vertical asymptotes over the interval $[-3\pi, 3\pi]$. (3 points)
- d) Find the x – intercepts (the zeros) over the interval $[-3\pi, 3\pi]$. (3 points)

Solution

a) No amplitude

$$b) f(-\pi) = -2 \tan\left(\frac{-2\pi}{3}\right) = 2 \tan\left(\frac{2\pi}{3}\right) = 2 \tan 120^\circ = -2 \tan 60^\circ = -2\sqrt{3}$$

$$c) \cos \frac{2x}{3} = 0 \Rightarrow \frac{2x}{3} = \frac{2n+1}{2} \pi, n \text{ is integer} \Rightarrow x = \frac{6n+3}{4} \pi$$

$$n = 0 \Rightarrow x = \frac{3\pi}{4}$$

$$n = 1 \Rightarrow x = \frac{9\pi}{4}$$

$$n = -1 \Rightarrow x = -\frac{3\pi}{4}$$

$$n = -2 \Rightarrow x = -\frac{9\pi}{4}$$

$$\therefore \text{V.A. are: } x = \pm \frac{3\pi}{4} \text{ and } x = \pm \frac{9\pi}{4}$$

$$d) \sin \frac{2x}{3} = 0 \Rightarrow \frac{2x}{3} = n\pi, n \text{ is integer} \Rightarrow x = \frac{3n\pi}{2}, n \text{ is integer}$$

$$n = 0 \Rightarrow x = 0$$

$$n = 1 \Rightarrow x = \frac{3\pi}{2}$$

$$n = 2 \Rightarrow x = 3\pi$$

$$n = -1 \Rightarrow x = -\frac{3\pi}{2}$$

$$n = -2 \Rightarrow x = -3\pi$$

$$\therefore x \text{ – intercepts are } (0,0), \left(\pm \frac{3\pi}{2}, 0\right) \text{ and } (\pm 3\pi, 0)$$