King Fahd University of Petroleum and Minerals
Faculty of Science – Math Prep Year program
Math 002 -042
Class Test

Name:	**Sr#:**	ID:	Sec.:

You must show all necessary steps for full mark

Question #	Points	Student's score
1	7	
2	18	
3	7	
4	9	
5	6	
6	12	
	Total	
		60

Question1 (4+3pts)

For the function $y = -2\csc(\frac{\pi}{2}x + \pi) + 1$

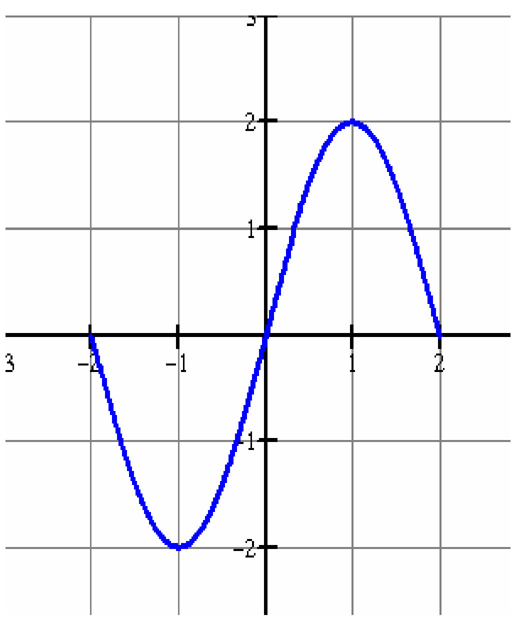
a) Complete the following table (**write the rule when that necessary**).

Solution:

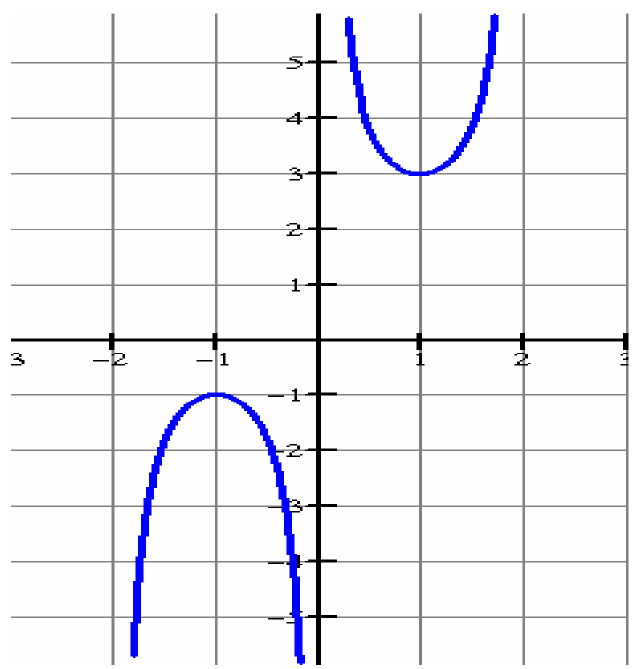
the period	the amplitude	the phase shift	the range
$p = \frac{2\pi}{\pi/2} = 4$ **1 point**	No amplitude **1 point**	$\frac{\pi}{2}x + \pi = 0$ $\Rightarrow x = -2$ (2 units left) **1 point**	$R = (-\infty, -\lvert a\rvert + d] \cup [\lvert a\rvert + d, \infty)$ $= (-\infty, -2+1] \cup [2+1, \infty)$ $= (-\infty, -1] \cup [3, \infty)$ **1 point**

b) Graph the function over one complete period

Solution:



$y = -2\sin(\frac{\pi}{2}x + \pi)$



$y = -2\csc(\frac{\pi}{2}x + \pi) + 1$

Question2 (18 pts)

Find the exact value of the following

a) $W(-\frac{13\pi}{3})$, where $W(t)$ is the wrapping function.

Solution:

$$W(-\frac{13\pi}{3}) = \left(\cos(-\frac{\pi}{3}), \sin(-\frac{\pi}{3})\right)$$
$$= \left(\cos\frac{\pi}{3}, -\sin\frac{\pi}{3}\right)$$
$$= \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

3 points

b) $(\csc x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$

Solution:

$$(\csc x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = (\frac{1}{\sin x} - \sin x)(\frac{1}{\cos x} - \cos x)(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x})$$
$$= (\frac{1-\sin^2 x}{\sin x})(\frac{1-\cos^2 x}{\cos x})(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x})$$
$$= (\frac{\sin^2 x}{\sin x})(\frac{\cos^2 x}{\cos x})(\frac{1}{\sin x \cos x})$$
$$= 1$$

3 points

c) $(\sin 15° + \cos 15°)^2$

Solution:

$$(\sin 15° + \cos 15°)^2 = \sin^2 15° + 2\sin 15° \cos 15° + \cos^2 15°$$
$$= (\sin^2 15° + \cos^2 15°) + 2\sin 15° \cos 15°$$
$$= 1 + \sin 30°$$
$$= 1 + \frac{1}{2} = \frac{3}{2}$$

3 points

d) $\cos^{-1}(\cos\frac{5\pi}{4})$

Solution:

$$\cos^{-1}(\cos\frac{5\pi}{4}) = \cos^{-1}(-\cos(\frac{\pi}{4}) = \cos^{-1}(-\frac{\sqrt{2}}{2}) = \pi - \cos^{-1}\frac{\sqrt{2}}{2} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

3 points

e) $\dfrac{\sin^{-1}x + \sec^{-1}\frac{1}{x}}{\cos^{-1}x + \sec^{-1}(-\frac{1}{x})}$

Solution:

$$\frac{\sin^{-1}x + \sec^{-1}\frac{1}{x}}{\cos^{-1}x + \sec^{-1}(-\frac{1}{x})} = \frac{\sin^{-1}x + \cos^{-1}x}{\cos^{-1}x + \cos^{-1}(-x)} = \frac{\frac{\pi}{2}}{\cos^{-1}x + \pi - \cos^{-1}x} = \frac{\frac{\pi}{2}}{\pi} = \frac{1}{2}$$

3 points

f) $\dfrac{1 + \tan(3x + \frac{2\pi}{3})\cot(\frac{\pi}{2} - 3x)}{\tan(3x + \frac{2\pi}{3}) - \cot(\frac{\pi}{2} - 3x)}$

Solution:

$$\frac{1 + \tan(3x + \frac{2\pi}{3})\cot(\frac{\pi}{2} - 3x)}{\tan(3x + \frac{2\pi}{3}) - \cot(\frac{\pi}{2} - 3x)} = \frac{1 + \tan(3x + \frac{2\pi}{3})\tan 3x}{\tan(3x + \frac{2\pi}{3}) - \tan 3x} = \cot(3x + \frac{2\pi}{3} - 3x) = \cot\frac{2\pi}{3} = -\cot 60° = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

3 points

Question3 (3+4 pts)

a) Verify that $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$.

Solution:

$$L.H.S. = \frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}} = \frac{\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \cos^2 x - \sin^2 x = \cos 2x = R.H.S.$$

3 points

b) **Use part a above** to solve the equation $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \dfrac{1}{2}, 0° \le x < 360°$.

Solution:

$$\cos 2x = \frac{1}{2} \Rightarrow 2\cos^2 x - 1 = \frac{1}{2} \Rightarrow 2\cos^2 x = \frac{3}{2} \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = 30° \quad or \quad x = 360° - 30° = 300°$$

$$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = 180° - 30° = 150° \quad or \quad x = 180° + 30° = 210°$$

$$S.S. = \{30°, 300°, 150°, 210°\}$$

4 points

Question4 (4+5 pts)

a) Find the domain and the range of the function $y = 2\cos^{-1}(x-1) - \pi$.

The Domain:

Solution:

$$-1 \le x - 1 \le 1$$
$$\Rightarrow 0 \le x \le 2$$
$$\therefore\ D = [0, 2]$$

2 points

The Range:

Solution:

$$0 \le \cos^{-1}(x-1) \le \pi$$
$$0 \le 2\cos^{-1}(x-1) \le 2\pi$$
$$0 - \pi \le 2\cos^{-1}(x-1) - \pi \le 2\pi - \pi$$
$$-\pi \le 2\cos^{-1}(x-1) - \pi \le \pi$$
$$\therefore\ R = [-\pi, \pi]$$

2 points

b) Solve the equation $\tan^{-1} x + \sin^{-1}(-\frac{4}{5}) = \cos^{-1}\frac{\sqrt{2}}{2}$

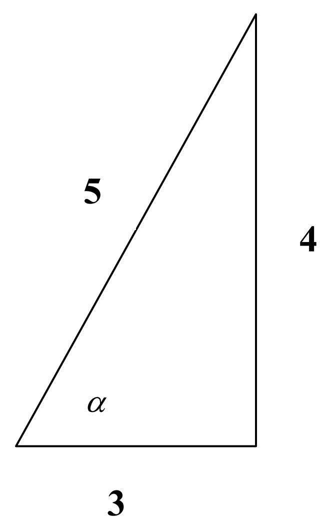
Solution:

$$\tan^{-1} x = \cos^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}(-\frac{4}{5})$$

$$\Rightarrow x = \tan\left(\cos^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}(-\frac{4}{5})\right)$$

$$\Rightarrow x = \tan\left(\frac{\pi}{4} - \alpha\right),\ let\,\alpha = \sin^{-1}(-\frac{4}{5}) \Rightarrow \sin\alpha = -\frac{4}{5}\ \ (\alpha \in 4^{th}\ Quadrant)$$

$$\Rightarrow x = \frac{\tan\frac{\pi}{4} - \tan\alpha}{1 + \tan\frac{\pi}{4}\tan\alpha} = \frac{1 + \frac{4}{3}}{1 - \frac{4}{3}} = \frac{\frac{7}{3}}{\frac{-1}{3}} = -7$$



5 points

Question5 (6Pts)

Determine which one of the following statements is TRUE **(T)** and which is FALSE **(F)** .Put the answer in the blank. **Justify your answer.**

a) If $\sin\frac{5\pi}{6}=\frac{1}{2}$, then $\sin^{-1}\frac{1}{2}=\frac{5\pi}{6}$. **F**

b) $f(x)=\frac{\sin x\tan x}{\cos x+\sec x}$ is an even function. **T**

c) $\cot^{-1}x=\frac{1}{\tan^{-1}x}$. **F**

d) The expression $\sin(-2°)\cos(-2°)\tan(-2°)\sec 2°\csc(-2°)\cot 2°$ is equal to -1. **T**

e) The expression $\frac{\sin^2 x-\cos^2 x}{1-\cos^2 2x}$ is equal to -1. **F**

f) $\frac{\sin 42°}{2}=\sin 21°\cos 21°$. **T**

Question6 (4+4+5 pts)

a) If $f(x) = -2\sqrt{3}\sin x\cos x + \cos 2x$, then write $f(x)$ in the form $f(x) = k\sin(2x+\alpha)$.

Solution:

$f(x) = -\sqrt{3}\sin 2x + \cos 2x$

$k = \sqrt{3+1} = 2$

$$\left.\begin{array}{l}\sin\alpha = \frac{1}{2}\\ \cos\alpha = -\frac{\sqrt{3}}{2}\end{array}\right\} \Rightarrow \alpha \in II\ Quadrant \Rightarrow \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}(or\ \ 150°)$$

4 points

$\therefore\ f(x) = 2\sin(2x + \frac{5\pi}{6})\ \ (\ Or\ \ f(x) = 2\sin(2x + 150°)\ \ )$

b) If $f(x) = 2\cot(3x - \frac{\pi}{3})$, then find the general equation of all **vertical asymptotes** for the graph of $f(x)$.

Solution:

$\sin(3x - \frac{\pi}{3}) = 0 \Rightarrow 3x - \frac{\pi}{3} = n\pi$, n is integer

$\Rightarrow 9x - \pi = 3n\pi$

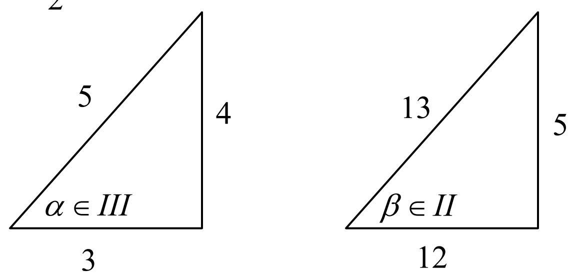
$\Rightarrow 9x = 3n\pi + \pi = (3n+1)\pi$

$\Rightarrow x = \frac{(3n+1)\pi}{9}$,

4 points

c) If $\sin\alpha = -\frac{4}{5}, \alpha$ in Quadrant III , and $\cos\beta = -\frac{12}{13}, \beta$ in Quadrant II, then find the exact value of $\cos^2(\frac{\alpha-\beta}{2})$.

Solution:



$$\cos^2(\frac{\alpha-\beta}{2}) = \frac{1+\cos(\alpha-\beta)}{2} = \frac{1+[\cos\alpha\cos\beta+\sin\alpha\sin\beta]}{2} = \frac{1+\left[(-\frac{3}{5})(-\frac{12}{13})+(-\frac{4}{5})(\frac{5}{13})\right]}{2} = \frac{1+\frac{16}{65}}{2} = \frac{81}{130}$$